Exercises for Physics for Physiotherapy Technology

203-9P1-DW Fall 2022

by Chris Roderick, Ph.D.

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Why practice?

Learning is about changing how you think. The most effective way to do that is to start by knowing how you think *now*. So always begin solving a problem by writing out what you think the answer will be like. Yes!, make a guess! But after that, do procede using the appropriate methods, and ***don't skip steps***. At the end, compare your result with your initial guess. Are they different? If yes, how do they differ? Are they different by a few percent? Or are they completely opposite?

That will be the place to pause and reflect on how you are thinking about these situations, and figure out what you need to change in your thinking. I'm here to help with that step, but it will go much faster if you contribute towards identifying where you need the help. Doing the exercises is the place where you work on that analysis.

When practicing problems spend the majority of your time being very explicit about the context, assumptions, and methods that you will be using in your process – that is, write everything out. This does take time, but it practices what is important: reasoning about the physics. You might not get as many problems done, but you will have done them better and gained more.

So now, let's get to work.

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Chapter 0

Preparation

0.1 Units

0.1.1 Lengths

Exr 0.1.01 1 m = ___ cm = _____ mm
Exr 0.1.02 1 cm = ____ m = __ mm
Exr 0.1.03 3.205 m = _____ cm = _____ mm
Exr 0.1.04 0.829 m = ____ cm = ___ mm
Exr 0.1.05 7 cm = ____ m = __ mm
Exr 0.1.06 237 cm = ____ m
Exr 0.1.07 15 mm = ___ cm = _____ m
Exr 0.1.08 29.45 cm = _____ mm = ______ m
Exr 0.1.09 14 in = _____ cm
Exr 0.1.10 42 in = ___ ft = _____ m
Exr 0.1.11 88 in = _____ m
Exr 0.1.12 1 ft = _____ cm
Exr 0.1.13 5’6” = 5 ft 6 in = ____ m
Exr 0.1.14 25 cm = ___ in
Exr 0.1.15 1.00 m = _ ft and ____ in
Exr 0.1.16 1.956 m = _ ft and _ in

0.1.2 Areas & Volumes

Exr 0.1.17 1 m^2 = ______ cm^2
Exr 0.1.18 400 cm^2 = ______ m^2
Exr 0.1.19 0.0820 m^2 = ___ cm^2
Exr 0.1.20 37 cm^2 = _____ mm^2
Exr 0.1.21 1 in^2 = ____ cm^2 = ___ mm^2
Exr 0.1.22 1 ft^2 = ___ in^2 = ___ cm^2
Exr 0.1.23 200 ft^2 = _____ m^2
Exr 0.1.24 50 cm^2 = ____ in^2
Exr 0.1.25 7 L = ____ cm^3 = _____ mL
Exr 0.1.26 375 mL = _____ L
Exr 0.1.27 0.520 L = ___ mL
Exr 0.1.28 1 m^3 = ________ cm^3 = _____ L
Exr 0.1.29 1 in^3 = _____ cm^3
Exr 0.1.30 1 ft^3 = ______ cm^3 = ____ L
Exr 0.1.31 10 L = _____ ft^3
Exr 0.1.32 625 mL = ____ in^3

0.1.3 Time

When working with measurements of time remember that (in this context) the non-standard symbols “d” for days, “h” for hours, and “min” for minutes are used. These are not to be confused with the Metric prefixes “d” for “deci” (10^{-1}), and “h” for “hecto” (10^{+2}). Be aware of the context.

For these conversions recall the definitions: $1\,\mathrm{d} = 24\,\mathrm{h}$, $1\,\mathrm{h} = 60\,\mathrm{min}$, and $1\,\mathrm{min} = 60\,\mathrm{s}$.

Exr 0.1.33 90 s = ___ min
Exr 0.1.34 250 s = ____ min
Exr 0.1.35 1 000 s = _____ min
Exr 0.1.36 15 s = ____ min
Exr 0.1.37 10.0 min = ____ s = _____ h
Exr 0.1.38 411 min = _________ s = ____ h
Exr 0.1.39 900 min = _________ s = ____ h
Exr 0.1.40 $\frac{1}{5}$ min = __ s = ___ h
Exr 0.1.41 0.20 h = __ min = _________ d
Exr 0.1.42 1.5 h = __ min = _____ d
Exr 0.1.43 8 h = ___ min = __ d
Exr 0.1.44 30 h = _____ min = ____ d
Exr 0.1.45 100 h = _____ min = ____ d
Exr 0.1.46 888 h = _________ min = __ d
Exr 0.1.47 $\frac{1}{10}$ d = ___ h = ___ min
Exr 0.1.48 $\frac{1}{3}$ d = _ h = ___ min
Exr 0.1.49 7 d = ___ h = ___ $\times 10^6$ s
Exr 0.1.50 30 d = ___ h = ___ $\times 10^6$ s
Exr 0.1.51 365 d = _____ h = ____ $\times 10^6$ s

0.1.4 Rates

Exr 0.1.52 1 m/s = __ m/min = _____ m/h
Exr 0.1.53 ____ m/s = 77 m/min = _____ m/h
Exr 0.1.54 ___ m/s = ___ m/min = 9 000 m/h
Exr 0.1.55 90 km/h = __ m/s
Exr 0.1.56 100 km/h = ____ m/s
Exr 0.1.57 10 m/s = __ km/h

Exr 0.1.58 343 m/s = _____ km/h
Exr 0.1.59 5.0 ft/s = __ km/h
Exr 0.1.60 50 mL/s = __ L/min
Exr 0.1.61 8.33 L/s = _____ m^3/min
Exr 0.1.62 2 L/min = ____ mL/s
Exr 0.1.63 0.370 m^3/min = ____ L/s

0.1.5 Masses & Forces

Exr 0.1.64 3.2 kg = _____ g
Exr 0.1.65 487 g = _____ kg
Exr 0.1.66 13 g = _____ kg
Exr 0.1.67 18 lb = ___ kg
Exr 0.1.68 145 lb = ____ kg

Exr 0.1.69 80 kg = ___ lb
Exr 0.1.70 72 kg weighs ___ N
Exr 0.1.71 ____ kg weighs 84.0 N
Exr 0.1.72 150 lb weighs ___ N
Exr 0.1.73 ___ lb weighs 750 N

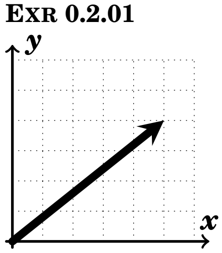
0.2 Vectors

0.2.1 Components, Magnitudes & Angles

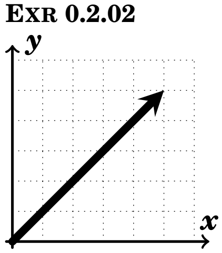
From Components to Magnitudes & Angles

For each of the vectors below find its components, then calculate its magnitude and the angle it makes with the $+x$-axis. (The angle measured counter-clockwise is positive.) In these exercises the grid size is 1 cm.

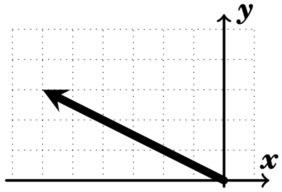
Exr 0.2.01



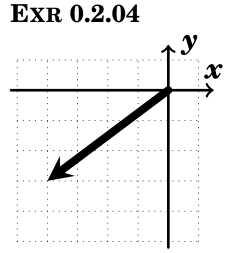
Exr 0.2.02



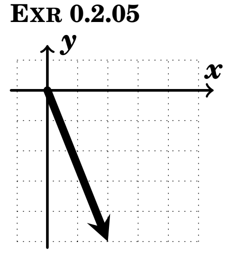
Exr 0.2.03



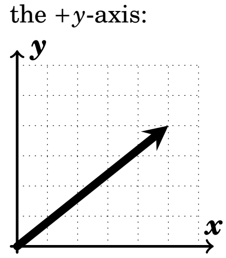
Exr 0.2.04



Exr 0.2.05



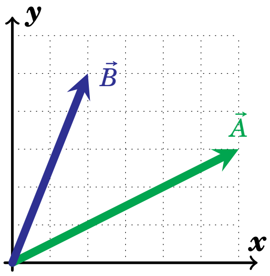
Exr 0.2.06 Find the angle between this vector and the $+y$-axis:



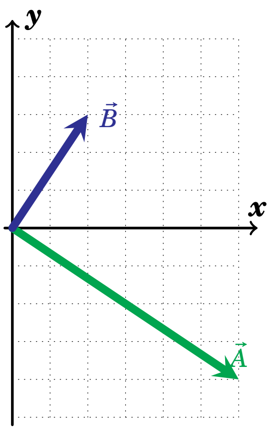
Angles Between Vectors

For each of the pairs of vectors below find the angle between them.

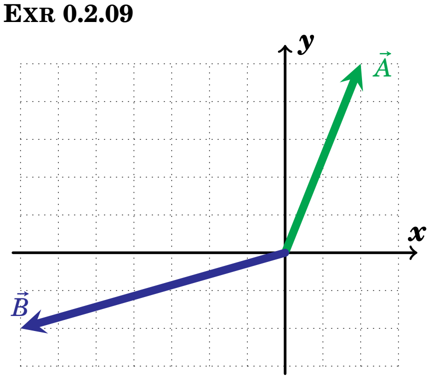
Exr 0.2.07



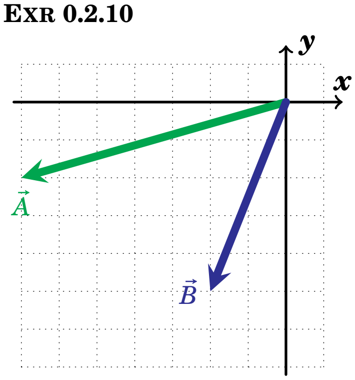
Exr 0.2.08



Exr 0.2.09



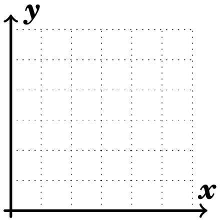
Exr 0.2.10



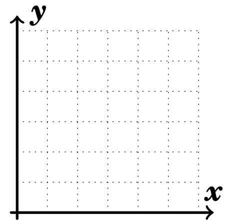
From Magnitudes & Angles to Components

For each of the vectors below find its components, then sketch it on the provided grid. (In these exercises the grid size is 1 cm.)

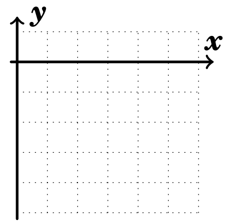
Exr 0.2.11 Draw the vector of magnitude 6.403 cm directed 38.7° counter-clockwise from the $+x$-axis.



Exr 0.2.12 Draw the vector of magnitude 6.708 cm directed 63.4° counter-clockwise from the $+x$-axis.



Exr 0.2.13 Draw the vector of magnitude 6.403 cm directed 51.3° clockwise from the $+x$-axis.



0.2.2 Sums of Vectors

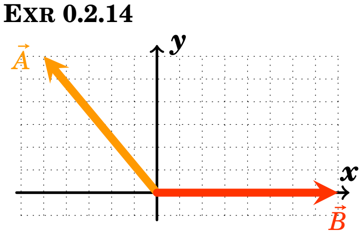
In this set of exercises we will practice the summation of vectors. In each exercise you will need to find the components of the vectors involved. Be explicit and careful with the units of the quantities you are using and calculating.

Sum of Pairs of vectors

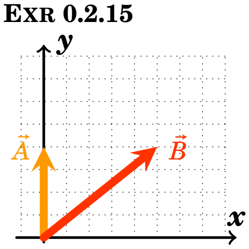
In the exercises below each vector is a position, and each grid square corresponds to one centimetre (1 cm) of distance.

For each of the pairs of vectors below calculate their sum. Find its components, its magnitude, and the angle it makes with the $+x$-axis. (The give vectors can be called $\vec{A}$ and $\vec{B}$. Call the sum $\vec{C} = \vec{A} + \vec{B}$.)

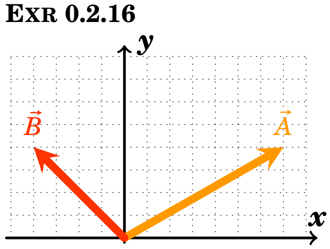
Exr 0.2.14



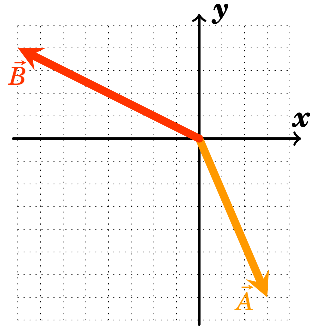
Exr 0.2.15



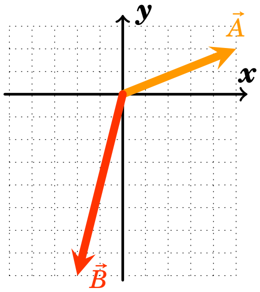
Exr 0.2.16



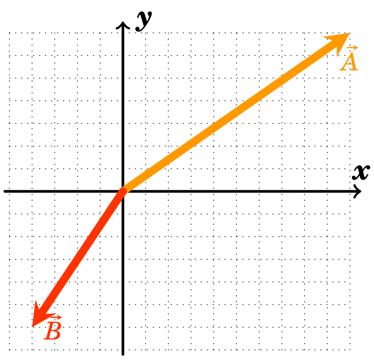
Exr 0.2.17



Exr 0.2.18



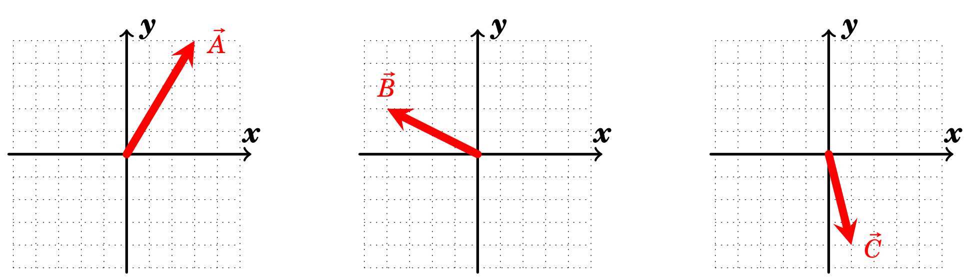
Exr 0.2.19



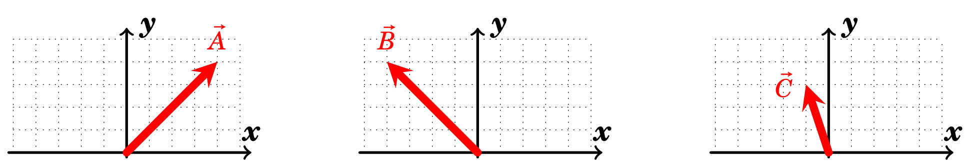
Comparing Sums

In the exercises below each vector is a force, and each grid square corresponds to one newton (1 N) of force. For the sets of vectors below find the pair of vectors whose sum (the resultant) has the greatest magnitude.

Exr 0.2.20



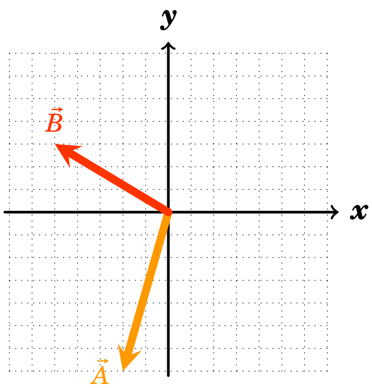
Exr 0.2.21



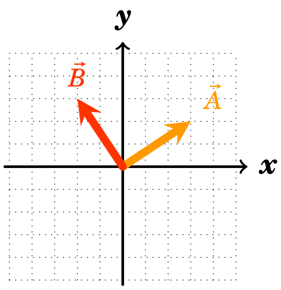
0.2.3 Vectors that Sum to Zero

In the exercises below each vector is a force, and each grid square corresponds to one newton (1 N) of force. For the sets of vectors below find the missing vector that would make their sum equal $\vec{0}$ N. (Remember that the vector $\vec{0}$ N is the vector whose components are each 0 N.) In the case of two vectors being given (which we can call $\vec{A}$ and $\vec{B}$) find the third vector $\vec{C}$ such that $\vec{A}+\vec{B}+\vec{C}=\vec{0}$ N.

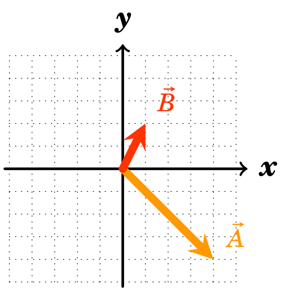
Exr 0.2.22



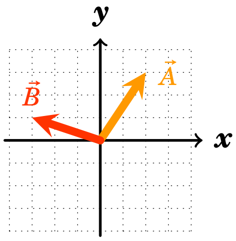
Exr 0.2.23



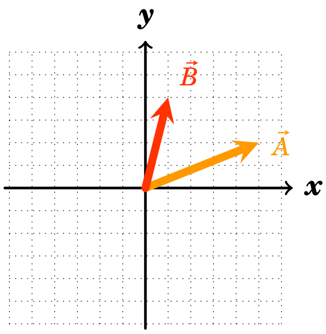
Exr 0.2.24



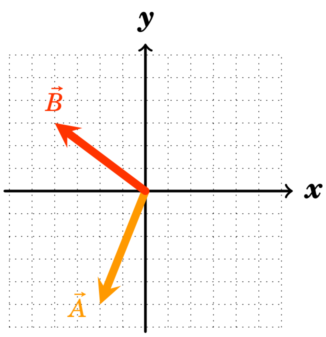
Exr 0.2.25



Exr 0.2.26



Exr 0.2.27



0.3 Logarithms

The square-root function tells you "if $x = \sqrt{y}$, then $y = x^2$." In a similar way the *logarithm* function tells you "if $x = \log(y)$, then $y = 10^x$." The logarithm (or just "log" for short) has some *algebraic* properties. You know that for the square-root $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$. For the log there are the rules $\log(a \times b) = \log(a) + \log(b)$ and $\log(c^n) = n \times \log(c)$. The exercises in this section practice these properties.

0.3.1 Basics

Exr 0.3.01 $\log(100) =$ _

Exr 0.3.02 $\log(1\,000) =$ _

Exr 0.3.03 $\log(500) =$ ____

Exr 0.3.04 $\log(50) =$ ____

Exr 0.3.05 $\log(5) =$ ___

Exr 0.3.06 $\log(1\,000\,000) =$ _

Exr 0.3.07 $\log(10\,000\,000) =$ _

Exr 0.3.08 $\log(3\,000\,000) =$ _____

Exr 0.3.09 $\log(9.900 \times 10^5) =$ _____

Exr 0.3.10 $\log(100 \times 1000) =$ _

Exr 0.3.11 $\log(100) + \log(1000) =$ _

Exr 0.3.12 $\log(15) =$ _____

Exr 0.3.13 $\log(3) + \log(5) =$ _____

Exr 0.3.14 $\log(1000/10) =$ _

Exr 0.3.15 $\log(1000) - \log(10) =$ _

Exr 0.3.16 $\log(15) =$ _____

Exr 0.3.17 $\log(30) - \log(2) =$ _____

Exr 0.3.18 $\log(7/3) =$ _____

Exr 0.3.19 $\log(7) - \log(3) =$ _____

Exr 0.3.20 $\log(3/7) =$ ______

Exr 0.3.21 $\log(137/137) =$ _

Exr 0.3.22 $\log(\frac{1}{100}) =$ ___

Exr 0.3.23 $\log(5) =$ ___

Exr 0.3.24 $\log(25) =$ ___

Exr 0.3.25 $2 \times \log(5) =$ ___

Exr 0.3.26 $\log(2) =$ ___

Exr 0.3.27 $\log(8) =$ ___

Exr 0.3.28 $3 \times \log(2) =$ ___

Exr 0.3.29 $\log(\sqrt{10}) =$ ___

Exr 0.3.30 $\log(\sqrt{\sqrt{10}}) =$ ___

0.3.2 Solving Equations with Logarithms

One last note: Since $10^x > 0$ for any value of x, there is no "±" when we use the logarithm to solve an equation. Also for that reason expressions like $\log(-7)$ have no meaning (just like $\sqrt{-7}$ is not a number).

Exr 0.3.31 If $10^x = 10^3$, then $x =$ ___.
Exr 0.3.32 If $10^x = 10^{-5}$, then $x =$ ___.
Exr 0.3.33 If $10^x = 10^7$, then $x =$ ___.
Exr 0.3.34 If $10^x = 10^{-2}$, then $x =$ ___.
Exr 0.3.35 If $10^x = 10^{3/4}$, then $x =$ ____.
Exr 0.3.36 If $10^x = 10^{-11/7}$, then $x =$ _____.
Exr 0.3.37 If $10^x = 10^{-37/81}$, then $x =$ ______.
Exr 0.3.38 If $10^x = 10^{5/2}$, then $x =$ ____.
Exr 0.3.39 If $10^x = 10^{2.957}$, then $x =$ ______.
Exr 0.3.40 If $10^x = 10^{\sqrt{2}}$, then $x =$ ____.
Exr 0.3.41 If $10^x = 10^{-0.8251}$, then $x =$ _______.
Exr 0.3.42 If $10^x = 10^{\pi}$, then $x =$ _.

Exr 0.3.43 If $\log(x) = +4$, then $x =$ ___________.
Exr 0.3.44 If $\log(x) = +1$, then $x =$ ________.
Exr 0.3.45 If $\log(x) = -5$, then $x =$ _____________.
Exr 0.3.46 If $\log(x) = -2$, then $x =$ _________.
Exr 0.3.47 If $\log(x) = -3/2$, then $x =$ _______________.
Exr 0.3.48 If $\log(x) = 4/11$, then $x =$ _______________.
Exr 0.3.49 If $\log(x) = 1\,\frac{1}{7}$, then $x =$ ________________.
Exr 0.3.50 If $\log(x) = 2.718282$, then $x =$ _________________.
Exr 0.3.51 If $\log(x) = \sqrt{3}$, then $x =$ _______________.
Exr 0.3.52 If $\log(x) = -\pi$, then $x =$ _________________.

Chapter 1

Forces

1.1 Free-Body Diagrams: Forces in Static Equilibrium

In these exercises we will practice the construction and use of Free-Body Diagrams (FBDs) to reason about the forces acting on objects in static equilibrium. For the purposes of practice we will limit ourselves to two-dimensional systems. Unless explicitly stated each system is being viewed from the side so that gravity acts straight downwards on the diagram.

In each exercise construct the Free-Body Diagram (FBD). A recommended first step is to make a list of the things that the object is interacting with. Name those things, beginning with the Earth, and then name the other objects or surfaces which the object is touching (like the floor) or is attached to (like a rope). Look carefully at the diagram (if given), and read carefully any descriptive text of the situation (if given), during this step.

After constructing the FBD, check that the forces do sum to zero. This check is a qualitative diagram used to confirm that equilibrium is *possible*. If it is not possible to sum the forces in your FBD by adjusting them, then return to your list of interactions, and think more carefully about the situation. (There is no point to move past this step to doing quantitative calculations if the required outcome is impossible!)

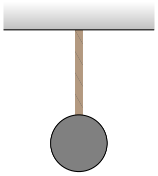
Remember to be clear about what aspect of each force is unknown. If there is a rope at a specified angle, then the force of tension that it exerts on the object is along that fixed direction, and only its magnitude may be adjusted. If there is a surface of contact, then the *normal* at that surface points perpendicular away from the surface, and only its magnitude may be adjusted. Similarly, if there is friction, then it must be parallel to the surface, but its direction and magnitude must both be determined.

Above all, do not hesitate to *iterate*. Producing the "correct" diagram immediately is not a healthy expectation to hold. You are solving a problem and you must be open to exploring alternatives before arriving at a consistent answer. Iteration is a key feature of problem-solving!

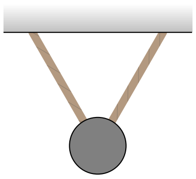
1.1.1 Gravity and Tension

In these first few Free-Body Diagram (FBD) exercises the object is suspended against gravity by ropes. In each exercise, draw the FBD, and use the sum of forces to *qualitatively* estimate the magnitudes of the tensions (relative to your choice of the weight of the object).

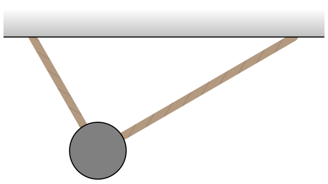
Exercise 1.1.01



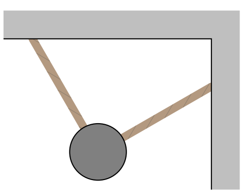
Exercise 1.1.02



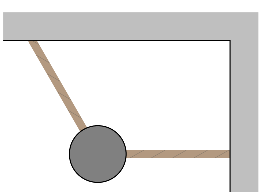
Exercise 1.1.03



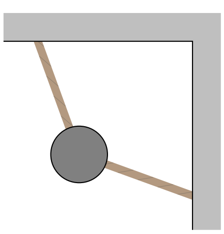
Exercise 1.1.04



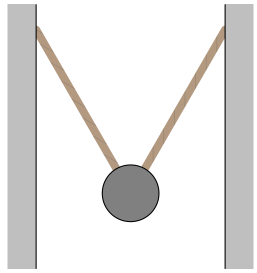
Exercise 1.1.05



Exercise 1.1.06



Exercise 1.1.07



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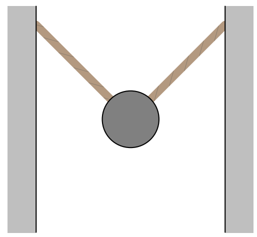
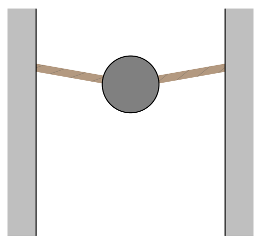
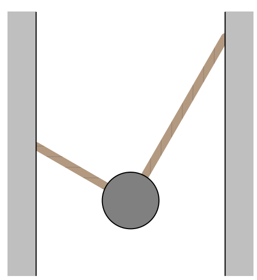


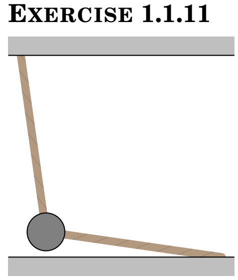
Exercise 1.1.09



Exercise 1.1.10



Exercise 1.1.11



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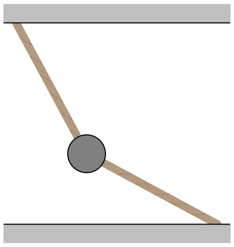
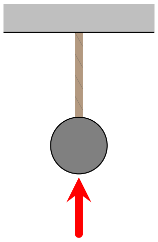


1.1.2 With an External Force

In these cases an external force $\vec{P}$ (a push or a pull) is being applied to an object suspended by ropes. In each of these cases the applied force never causes the tension of any rope to become zero.

Exercise 1.1.13



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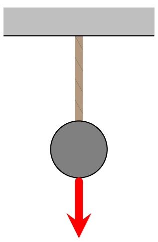

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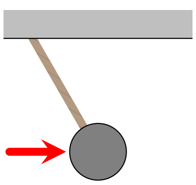
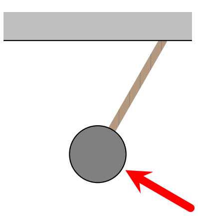


Exercise 1.1.16



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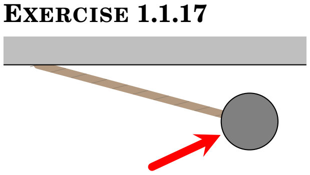

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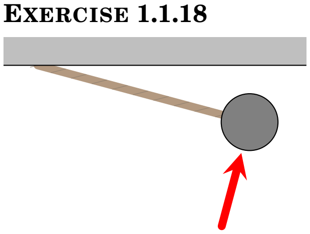
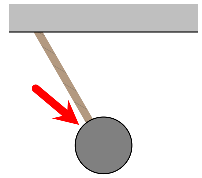
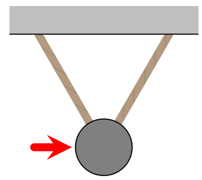


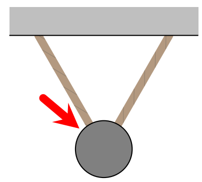
Exercise 1.1.19



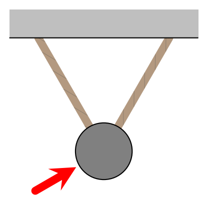
Exercise 1.1.20



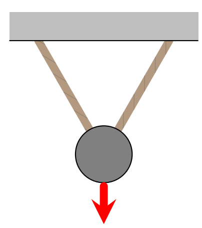
Exercise 1.1.21



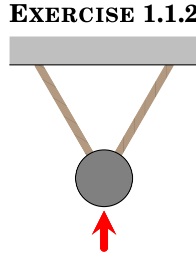
Exercise 1.1.22



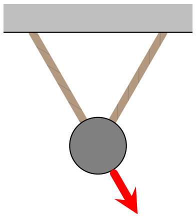
Exercise 1.1.23



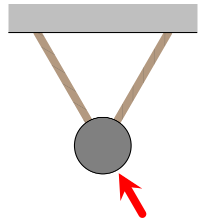
Exercise 1.1.24



Exercise 1.1.25



EXERCISE 1.1.26



1.1.3 Gravity and Contact

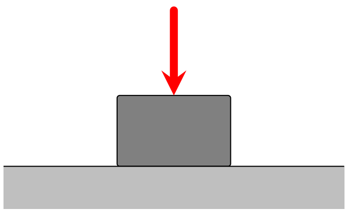
In this section we will practice analyzing objects in contact with a surface. Friction is present between all surfaces of contact. The *exceptions* of there being negligible friction will be noted as $\mu = 0$.

Level Surfaces

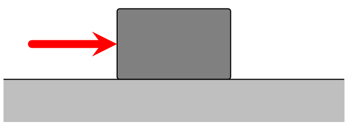
In these situations the force of gravity and the *normal* will point opposite each other, but, because of other forces present, they may not be of equal magnitude. Careful!

EXERCISE 1.1.27

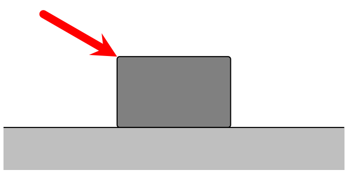
EXERCISE 1.1.28



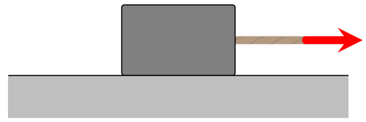
EXERCISE 1.1.29



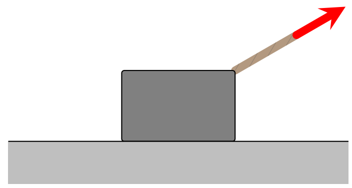
EXERCISE 1.1.30



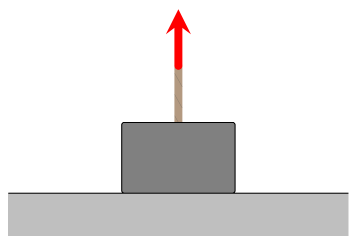
EXERCISE 1.1.31



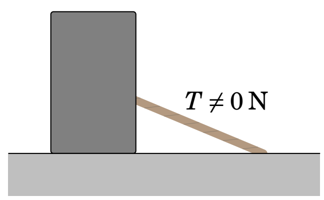
EXERCISE 1.1.32



EXERCISE 1.1.33



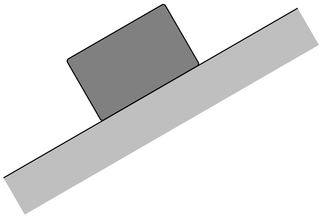
EXERCISE 1.1.34 An object is attached to the surface by a rope. There is tension in the rope.



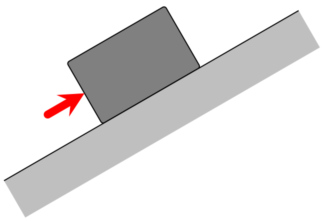
Inclined Surfaces

In these exercises friction remains strong enough to keep the object in equilibrium on the surface. The *normal* is still (as always) perpendicular to the surface, but with the surface not horizontal the normal will *not* point opposite gravity, and will almost certainly not have the same magnitude. Be very careful finding the normal!

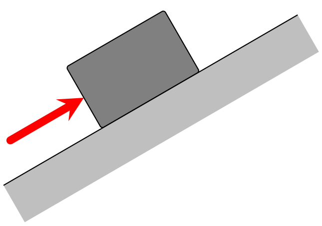
EXERCISE 1.1.35



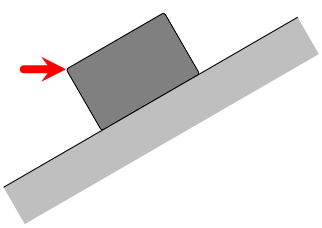
EXERCISE 1.1.36 The applied force is small in comparison to all other forces.



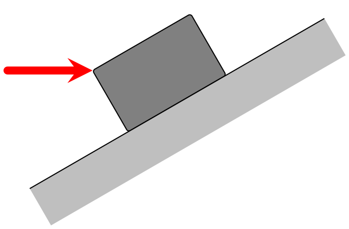
EXERCISE 1.1.37 The applied force is large, and the object almost starts sliding UP the incline.



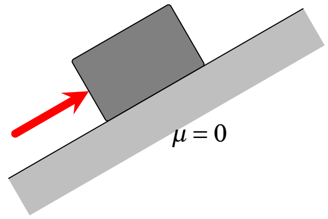
EXERCISE 1.1.38 The applied force is small in comparison to all other forces.



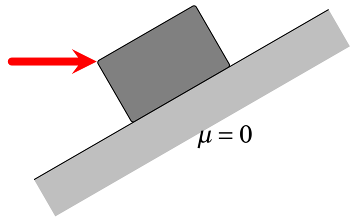
EXERCISE 1.1.39 The applied force is large, and the object almost starts sliding UP the incline.



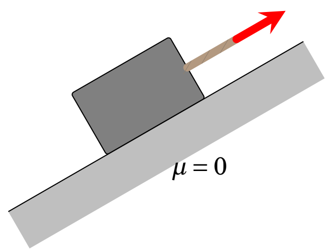
EXERCISE 1.1.40



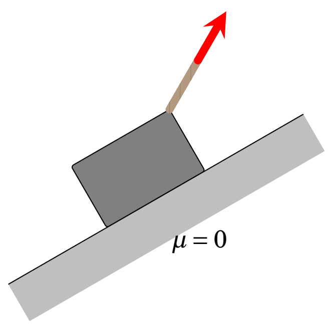
EXERCISE 1.1.41



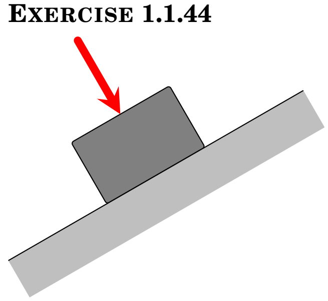
EXERCISE 1.1.42



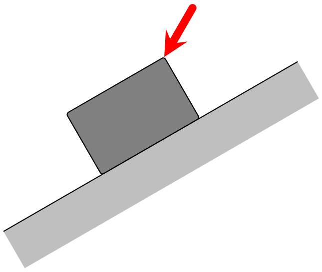
EXERCISE 1.1.43



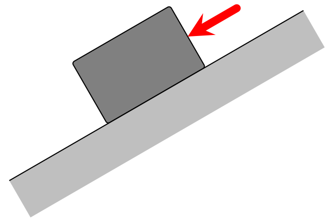
EXERCISE 1.1.44



EXERCISE 1.1.45



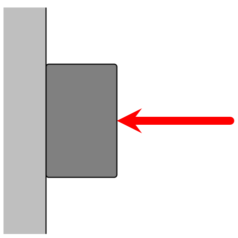
Exercise 1.1.46



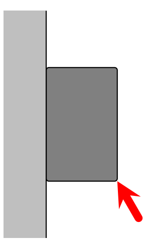
Cases of vertical and inverted surface

The theme to recognize in these exercises is that, with the surface vertical or even *upside-down*, the normal force can not contribute to supporting the object's weight, and may in fact be contributing a downwards component! Look to the applied force(s) and friction (when present) to support the object against gravity.

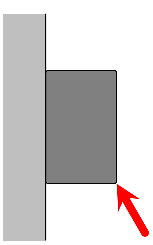
Exercise 1.1.47 The applied force has a magnitude *much larger* than the block's weight.



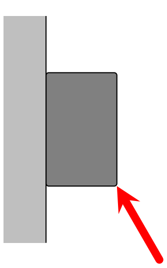
Exercise 1.1.48 The applied force has a magnitude *smaller* than the block's weight.



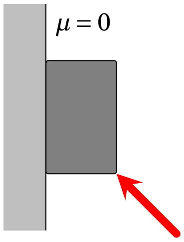
Exercise 1.1.49 The applied force has a magnitude *equal* to the block's weight.



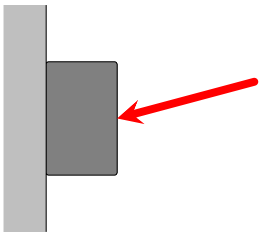
Exercise 1.1.50 The applied force has a magnitude *larger* than the block's weight.



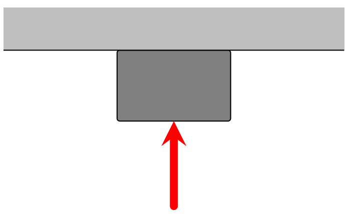
Exercise 1.1.51 No friction between the block and surface.



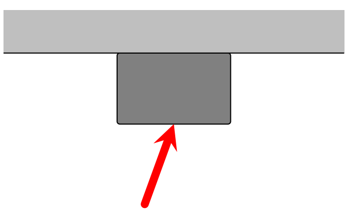
EXERCISE 1.1.52 The applied force has a magnitude *much larger* than the block's weight.



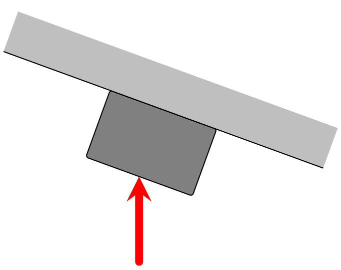
EXERCISE 1.1.53 The applied force has a magnitude *larger* than the block's weight.



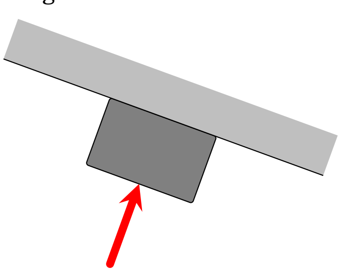
EXERCISE 1.1.54 The applied force has a magnitude *larger* than the block's weight.



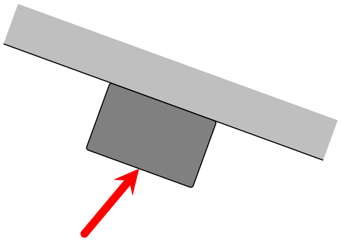
EXERCISE 1.1.55 The applied force is vertically upwards, and has a magnitude *larger* than the block's weight.



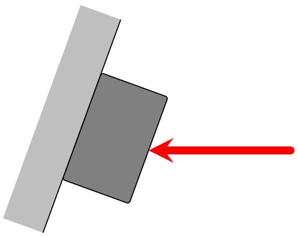
EXERCISE 1.1.56 The applied force is perpendicular into the surface, and has a magnitude *larger* than the block's weight.



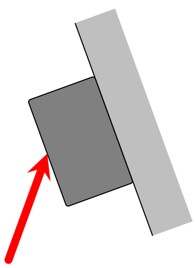
EXERCISE 1.1.57 The applied force is pointed slightly *down* the incline, and has a magnitude *larger* than the block's weight.



EXERCISE 1.1.58 The applied force is horizontal, and has a magnitude *much larger* than the block's weight.



EXERCISE 1.1.59 The applied force has a magnitude *larger* than the block's weight.

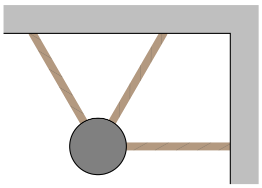


1.1.4 Indeterminate Problems

It is possible for there to be more unknown forces than there are are equations. These cases are called *indeterminate*. The possible solutions are subdivided into *cases* categorized by assumed values for one (or more) of the unknowns.

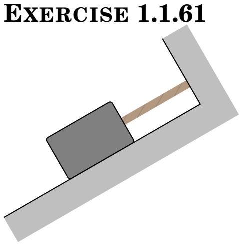
In these exercises there is friction between the ropes and any surfaces that they lay across. This means that, in any cases where a segment of rope lays on a surface, the tension may vary along that length of the rope!

EXERCISE 1.1.60



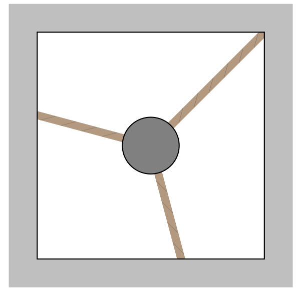
Solve the system (above) for the three unknown tensions in these cases:
(1) The tension in the horizontal rope is zero.
(2) The tension in the rope that points upwards to the right is zero.
(3) The tension in the rope that points upwards to the left equals the object's weight mg.

EXERCISE 1.1.61

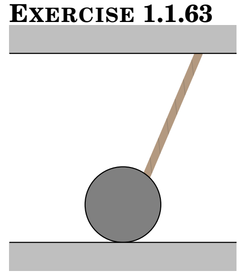


Solve the system (above) for the unknown tension in these cases:
(1) The tension in the rope is very small, but not zero.
(2) The tension in the rope is a value that lets the friction be zero.
(3) The tension in the rope equals the object's weight mg.

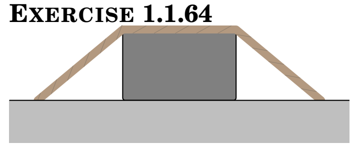
EXERCISE 1.1.62



EXERCISE 1.1.63

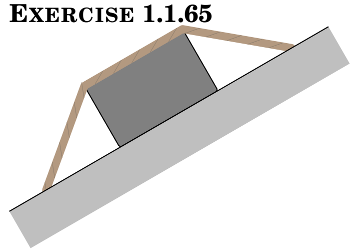


EXERCISE 1.1.64



There is friction between the top of the block and rope. This means that the value of the tension in the rope can be different along its length where lays across the top of the block! Consequently the tension of the segment on the left (from the block to the surface) can be different from the the tension of the segment on the right.

EXERCISE 1.1.65



1.2 Solving problems using *the Process*

The Process

0. Identify the Object!

1. Identify the forces acting on the Object.

2. Draw the Free-Body Diagram.

3. Separately, for each force acting on the Object:

- draw the coordinates
- draw the force (vector)
- determine the components.

4. Use Newton's 1st Law to write the equations to be solved.

5. Solve the equations for the unknown quantities.

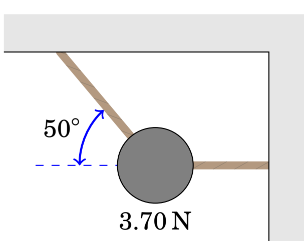
Remember to take your time and to work carefully through Steps 0, 1 and 2 – that's where you are doing the physics! Working on these problems is not a race. If you rush through (or skip) those steps, then you will risk getting it *completely wrong*. Working on these problems is where you should practice working methodically through each step of the Process.

1.2.1 Mechanical systems

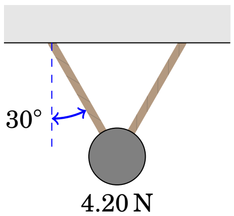
These Problems are all mechanical (not BIO-mechanical) in that there are no humans or factors of human anatomy involved.

PROBLEM 1.2.01: On the force table a 250 gram mass is hanging at 72° and a 300 gram mass is hanging at 345°. What mass must we hang (and at what direction) to keep the ring at the center in static equilibrium? (The weight of the ring may be ignored since it is much smaller than the tension in the strings will be.)

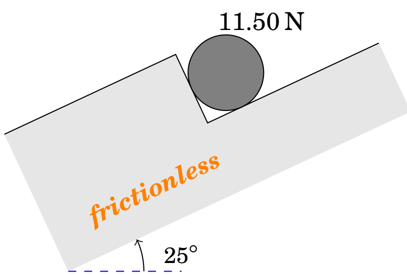
PROBLEM 1.2.02: An object of weight 3.70 N is hanging by two ropes, as shown. Find the magnitude of the tension in each rope.



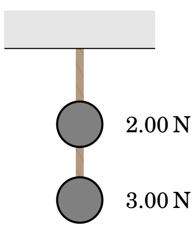
Problem 1.2.03: An object of weight 4.20 N is hanging from two ropes that are symmetric (as shown). Find the magnitude of the tension in each rope.



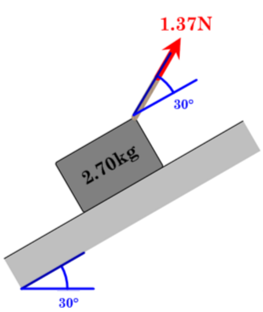
Problem 1.2.04: A spherical object of weight 11.50 N rests in a corner of a frictionless surface. Find the magnitude of the normal exerted by each surface.



Problem 1.2.05: Two objects are hanging as shown. Find the magnitude of the tension in each rope.



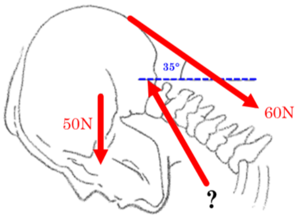
Problem 1.2.06: An object remains at rest on an incline when a force is applied as shown in the diagram. The mass of the object is 2.70 kg and the magnitude of the applied pull is 1.37 N. What is the magnitude of the normal acting on the object? What is the magnitude and direction of the static friction acting on the object?



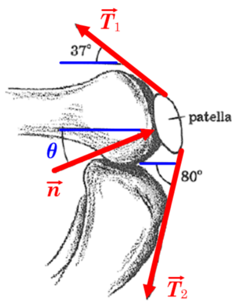
1.2.2 Biomechanical systems

These Problems are all biomechanical in that there are humans or factors of human anatomy involved.

PROBLEM 1.2.07: A student looking at their cellphone has their head bent over, as shown. The weight of their head is 50N, and the tension in the muscles attached to the base of their skull is 60 N. What force is exerted by the 1st cervical vertebrae onto the base of their skull? (Specify magnitude and direction.)

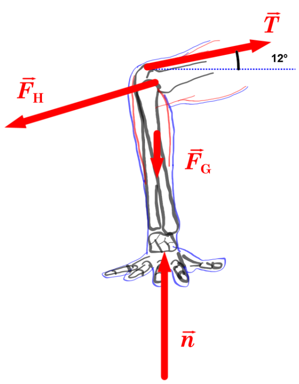


PROBLEM 1.2.08: The tendon from the quadraceps muscle (thigh) passes over the patella (kneecap) to attach to the tibia (shin bone). The tension in the tendon is 1333N. What are the magnitude and direction of the contact force between the patella and the femur (thigh bone)? (There is essentially no friction between the patella and femur. The mass of the patella is only a few grams, so gravity may be ignored relative to the tension in the tendon.)

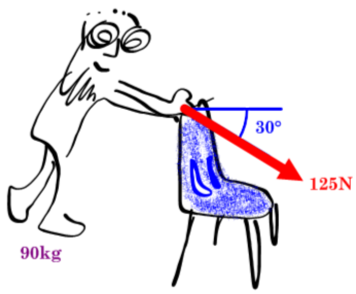


PROBLEM 1.2.09: A person is doing a push-up. Choosing the right forearm (including the hand) as the Object, we want to find the force acting at the elbow joint where the bones of the upper and lower arm meet. There is the contact force $\vec{n}$ with the floor (magnitude 200.0 N) that acts vertically upwards. There is, of course, the weight $\vec{F}_G$ of the forearm itself (86.8 N), acting vertically downwards. The tension $\vec{T}$ in the triceps muscle, attached to the ulna, exerts a force of 1 826.6 N along a direction of 12° above the horizontal.

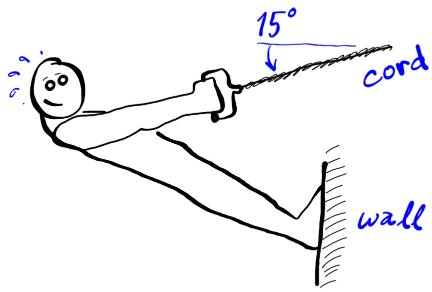
Find the magnitude and the direction of the force $\vec{F}_H$ exerted by the humerus bone (of the upper arm) onto the ulna bone (of the forearm).



PROBLEM 1.2.10: As I push a chair away from me (as shown below) what are the normal and friction forces at my feet if I do not slide while pushing?



PROBLEM 1.2.11: An acrobat of weight 700 N is practicing a maneuver, suspending themselves from a vertical wall, as shown in the picture. The rope suspending them makes an angle of 15° with the horizontal, and has a tension of 1160 N. What is the magnitude and direction of the friction exerted on the acrobat's feet by the wall.



1.3 Pulleys

A pulley is a machine that changes the *direction* of a rope but not the tension. The following exercises investigate the mechanical consequences of that fact. The context, as always, is static equilibrium.

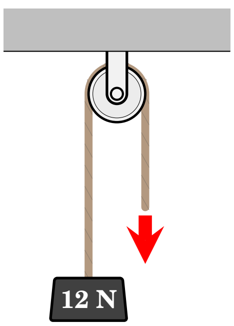
In each of the exercises below find the tension in each rope, when possible. Where asked, find the unknown externally applied force (appearing as a red vector in the diagrams).

To solve for the tension trace along the rope and treat each pulley as if it was its own object. Each pulley being in static equilibrium means that the sum of forces acting on each pulley must sum to zero, individually.

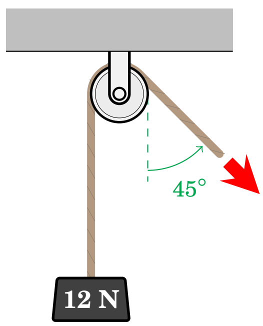
Pay very close attention to how the system of pulleys attaches to weights in the problem; you will quite often find that the tension in the rope is only a fraction of the object's weight.

1.3.1 A Single Pulley

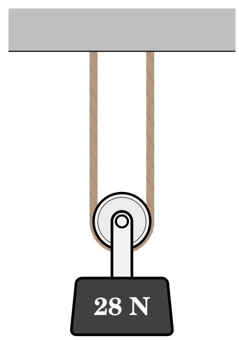
Exr 1.3.01



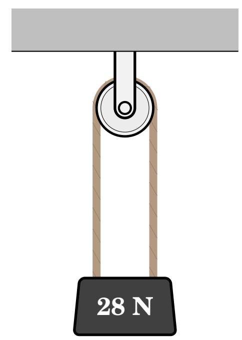
Exr 1.3.02



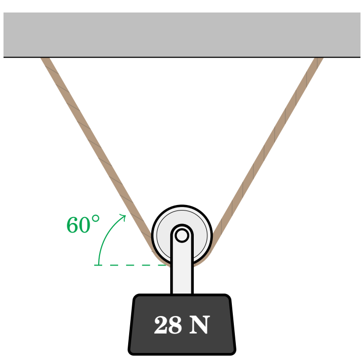
Exr 1.3.03



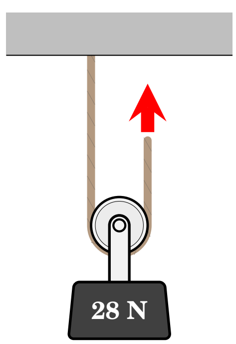
Exr 1.3.04



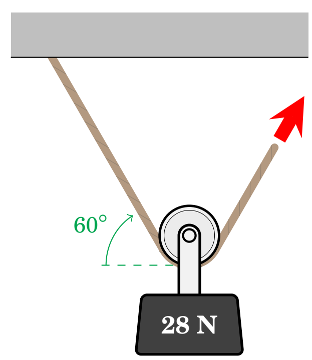
Exr 1.3.05



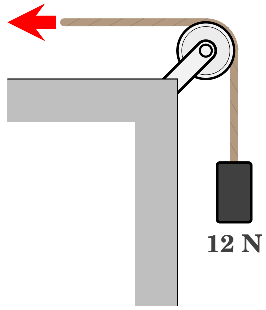
Exr 1.3.06



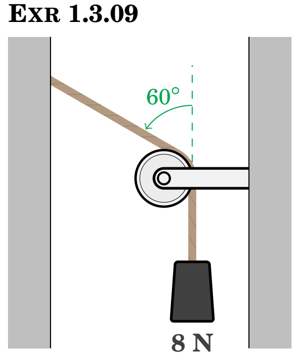
Exr 1.3.07



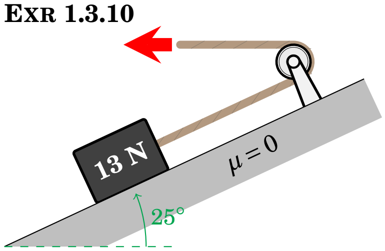
Exr 1.3.08



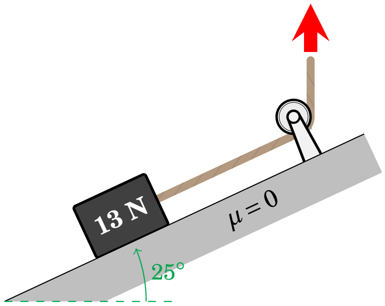
Exr 1.3.09



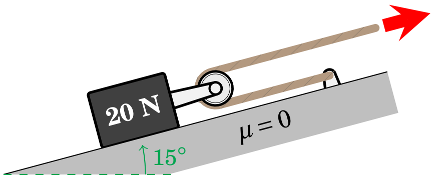
Exr 1.3.10



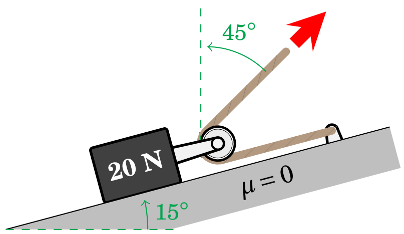
Exr 1.3.11



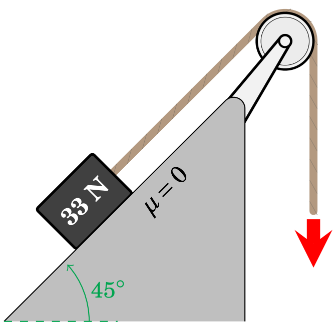
Exr 1.3.12



Exr 1.3.13

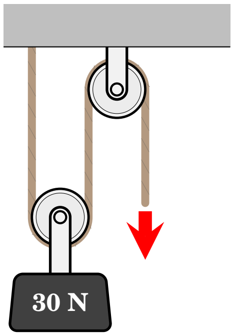


Exr 1.3.14

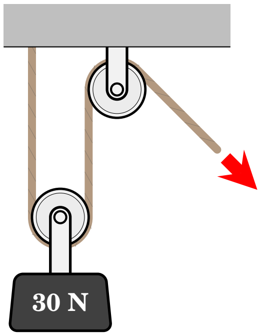


1.3.2 Two Pulleys

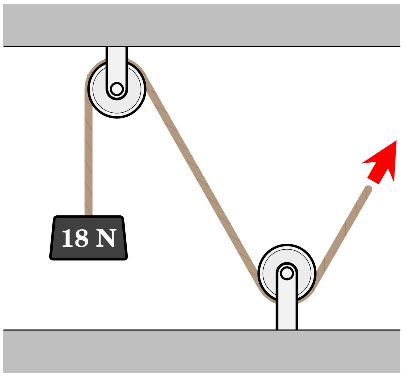
Exr 1.3.15



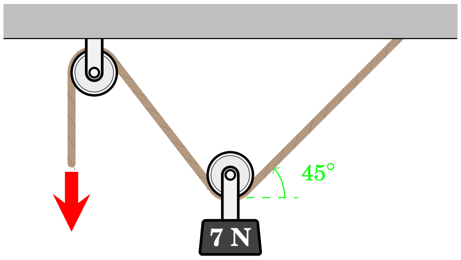
Exr 1.3.16



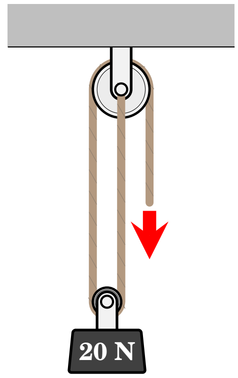
Exr 1.3.17



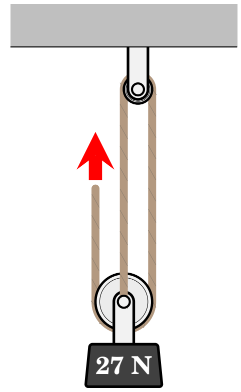
Exr 1.3.18



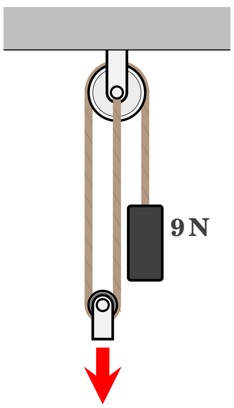
Exr 1.3.19



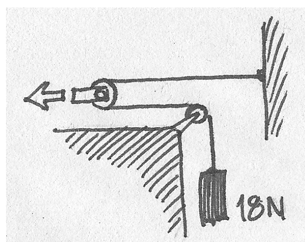
Exr 1.3.20



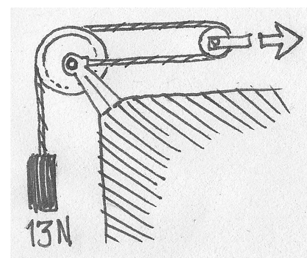
Exr 1.3.21



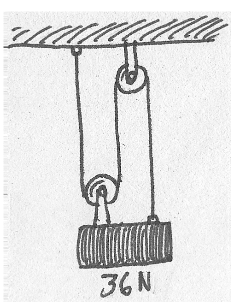
Exr 1.3.22



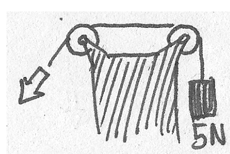
Exr 1.3.23



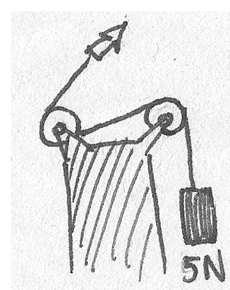
Exr 1.3.24



Exr 1.3.25

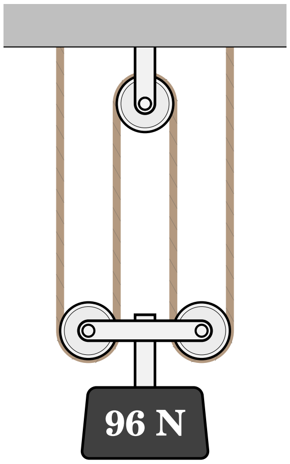


Exr 1.3.26

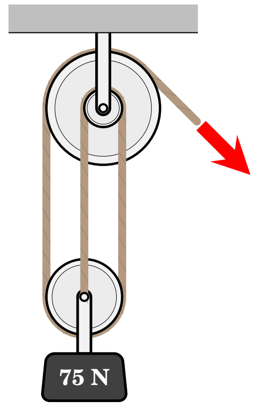


1.3.3 Three or more Pulleys

Exr 1.3.27



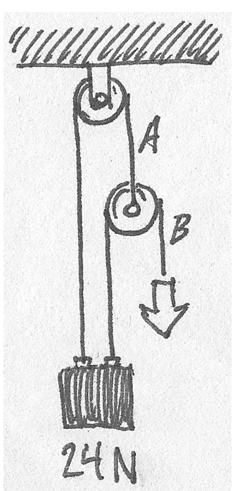
Exr 1.3.28



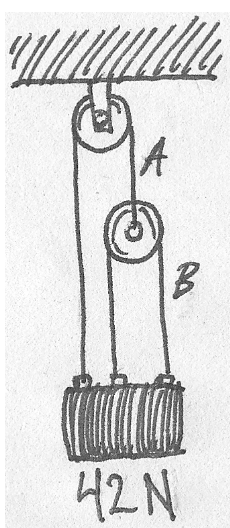
1.3.4 Multiple Ropes

In each of these systems find the tension in each of the ropes.

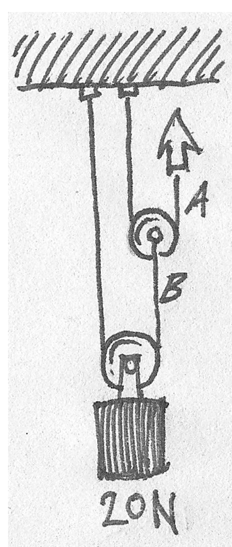
Exr 1.3.29



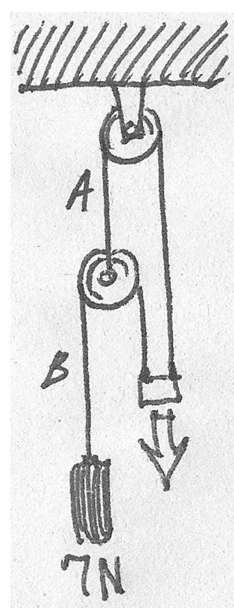
Exr 1.3.30



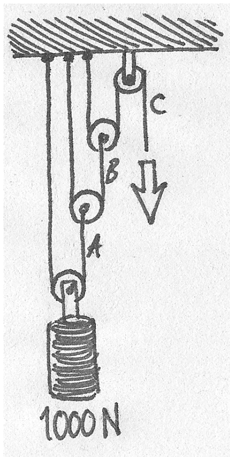
Exr 1.3.31



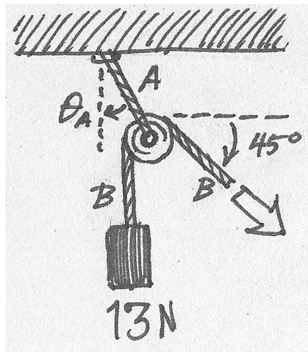
Exr 1.3.32



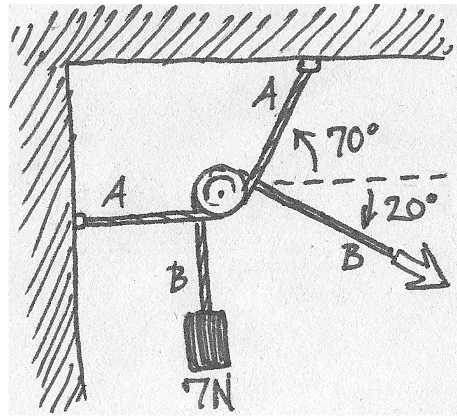
Exr 1.3.33



Exr 1.3.34



Exr 1.3.35



Chapter 2

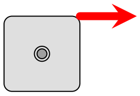
Torques

2.1 Torque : Qualitative Exercises

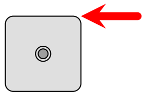
2.1.1 Sign of Torque

In each of the following exercises find the *sign* of the torque about the pivot produced by the single applied force. If the applied force produces a torque of magnitude zero, say so. Remember that the sign of the torque is the sign of the rotation that would happen *if* that torque was the only one being applied. (The sign of rotation follows the sign convention of angles, with counter-clockwise being positive and clockwise being negative.)

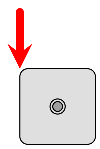
Exr 2.1.01



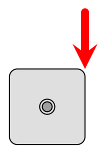
Exr 2.1.02



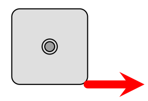
Exr 2.1.03



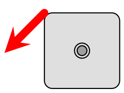
Exr 2.1.04



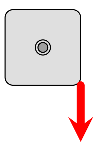
Exr 2.1.05



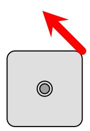
Exr 2.1.06



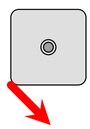
Exr 2.1.07



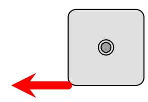
Exr 2.1.08



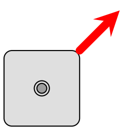
Exr 2.1.09



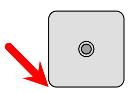
Exr 2.1.10



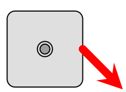
Exr 2.1.11



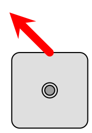
Exr 2.1.12



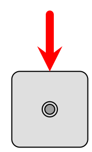
Exr 2.1.13



Exr 2.1.14



Exr 2.1.15



2.1.2 Line of Action

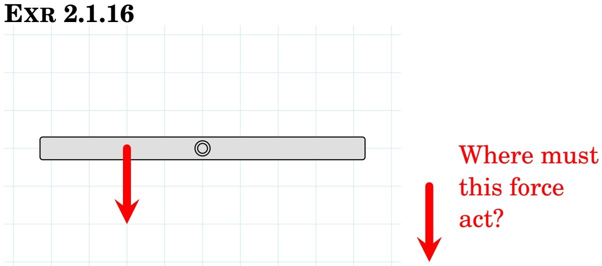
In the following exercises we use the *line of action* to reason qualitatively about the magnitude of torque. The context is static equilibrium, in which $\sum \vec{\tau} = \vec{0}\,\mathrm{N{\cdot}m}$ about any axis. With the object in the xy-plane (the page), the z-axis will be through the chosen pivot (perpendicular to the page), and $\sum \tau_z = 0\,\mathrm{N{\cdot}m}$ about that axis.

In the cases where we are given a force, the exercise is to find the line of action along which it must act so that the object remains in static equilibrium. In the cases where we are given a line of action, the exercise is to find (qualitatively) the force that would, acting along that line, keep the object in static equilibrium.

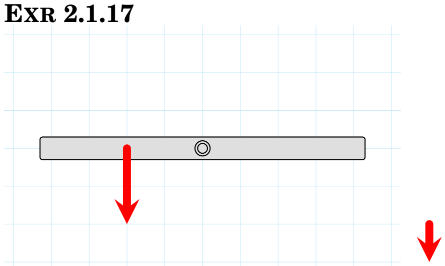
Thin rectangular rod

In this first set of exercises the pivot will be located at the center of gravity of the object. Neither the force of gravity nor the contact force of the pivot contribute a torque, so their magnitude and direction are unimportant. The forces do still exist but, to simplify the diagrams, neither will be shown.

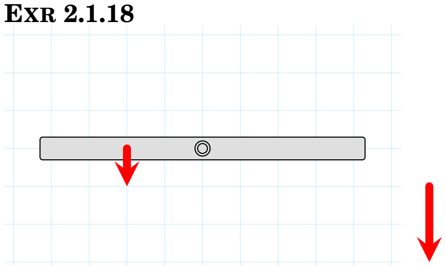
Exr 2.1.16



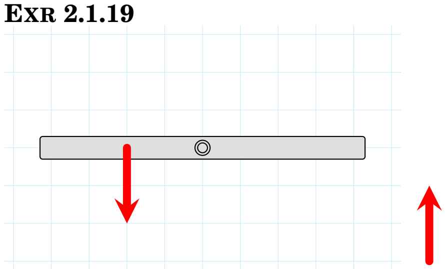
Exr 2.1.17



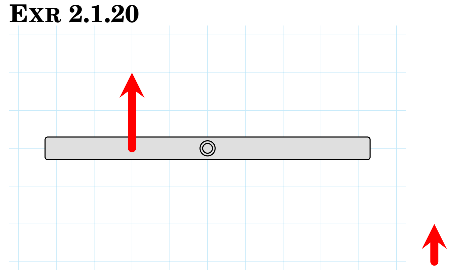
Exr 2.1.18



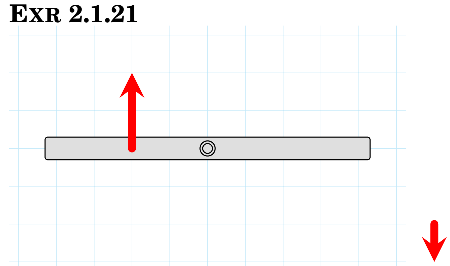
Exr 2.1.19



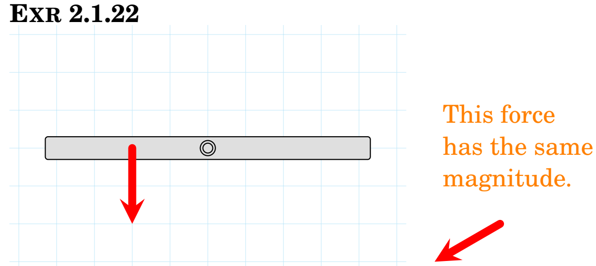
Exr 2.1.20



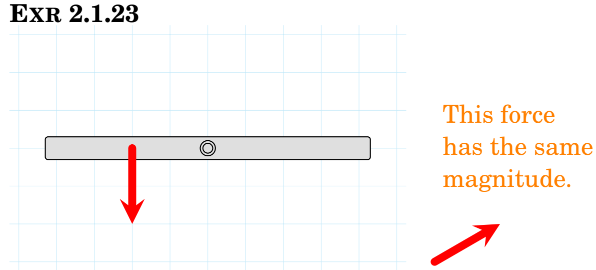
Exr 2.1.21



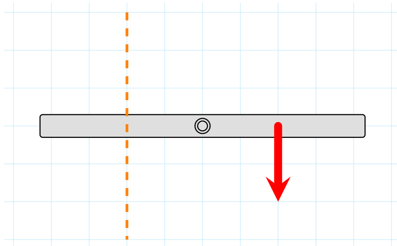
Exr 2.1.22



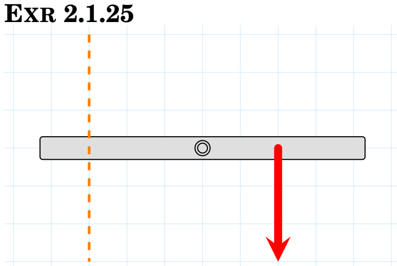
Exr 2.1.23



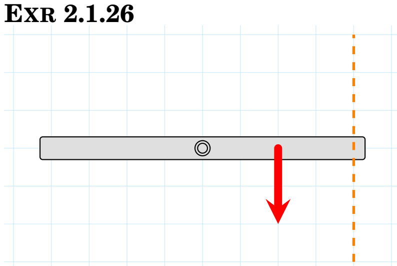
Exr 2.1.24
What force must be placed on the dashed orange line to maintain static equilibrium?



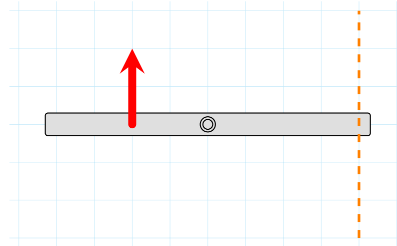
Exr 2.1.25



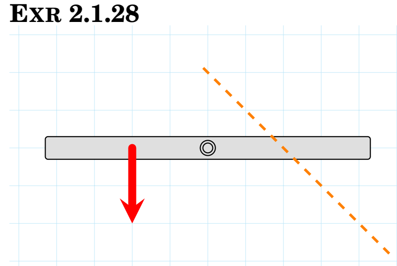
Exr 2.1.26



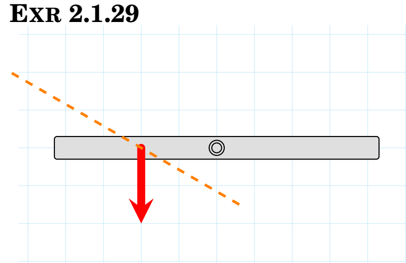
Exr 2.1.27



Exr 2.1.28

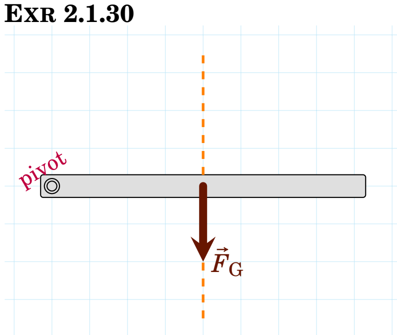


Exr 2.1.29

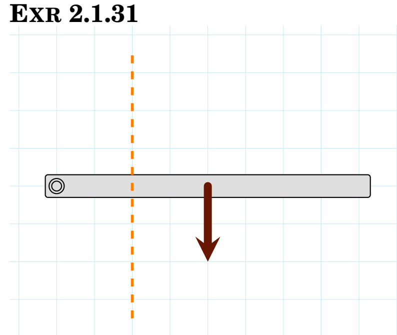


In this next set of exercises the pivot is ***not*** at the center of gravity of the object. This means that gravity will exert a torque about the pivot on the object. (The force of gravity $\vec{F}_G$ is coloured dark brown in the diagrams.) The goal will be to find or place the force that will maintain equilibrium.

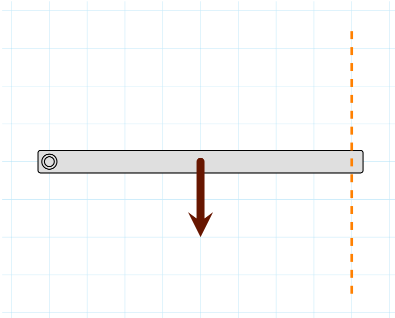
Exr 2.1.30



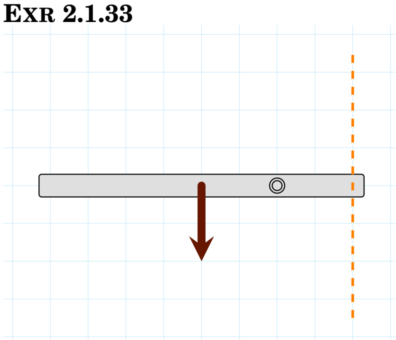
Exr 2.1.31



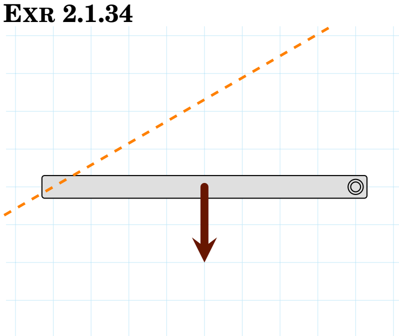
Exr 2.1.32



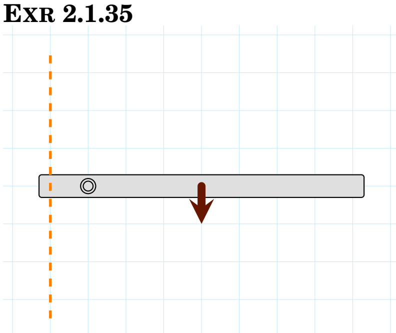
Exr 2.1.33



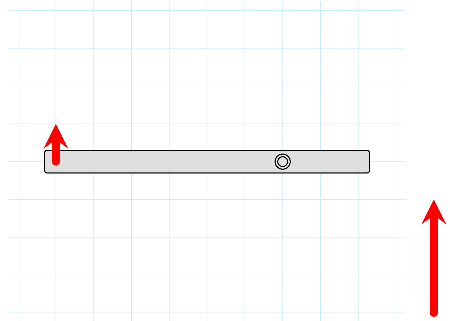
Exr 2.1.34



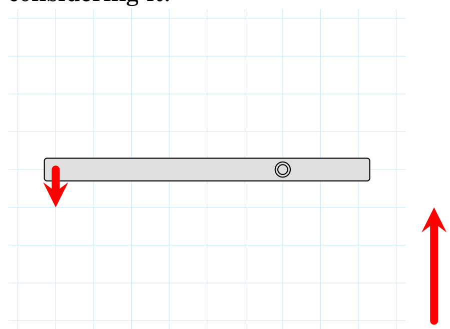
Exr 2.1.35



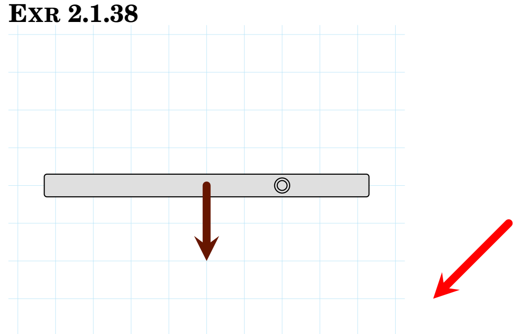
Exr 2.1.36
In this exercise gravity is present, but is so much smaller than the other forces present that we omit considering it.



Exr 2.1.37
In this exercise gravity is present, but is so much smaller than the other forces present that we omit considering it.

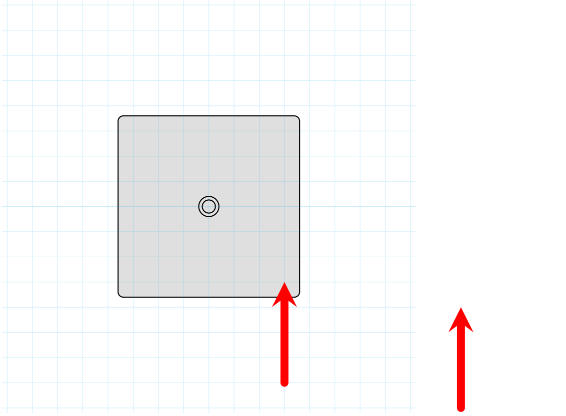


Exr 2.1.38

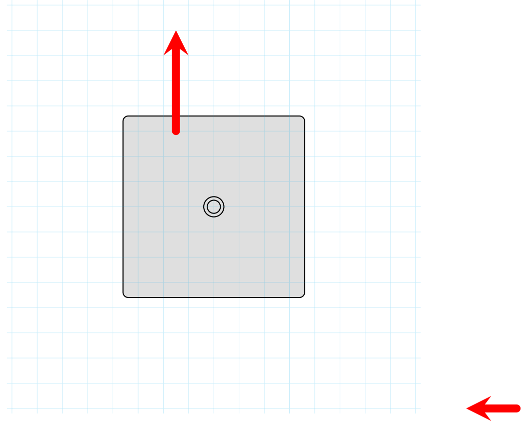


Square and rectangular objects

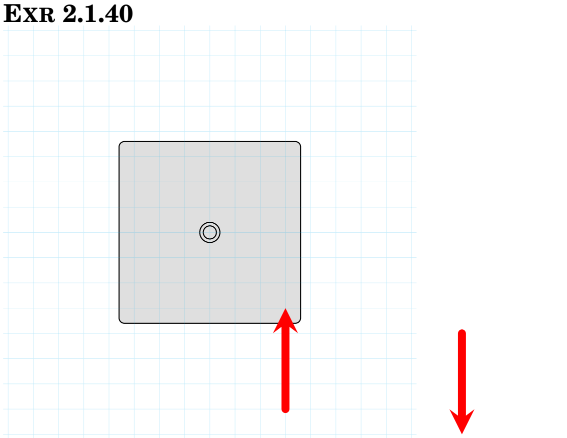
Exr 2.1.39



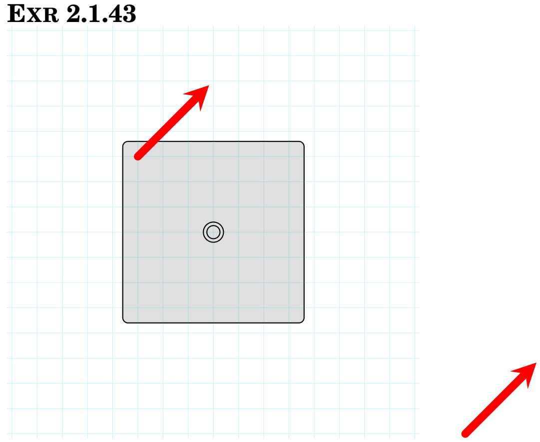
Exr 2.1.42



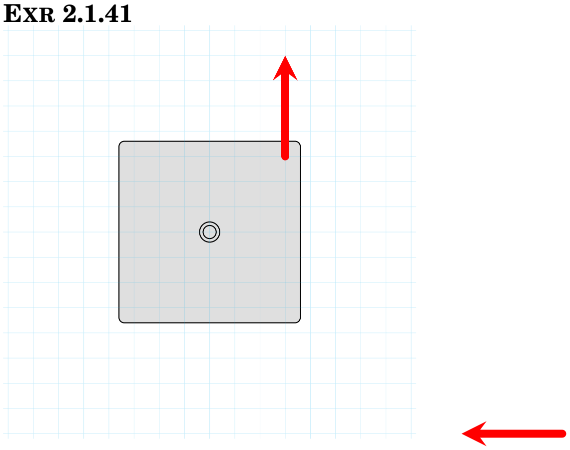
Exr 2.1.40



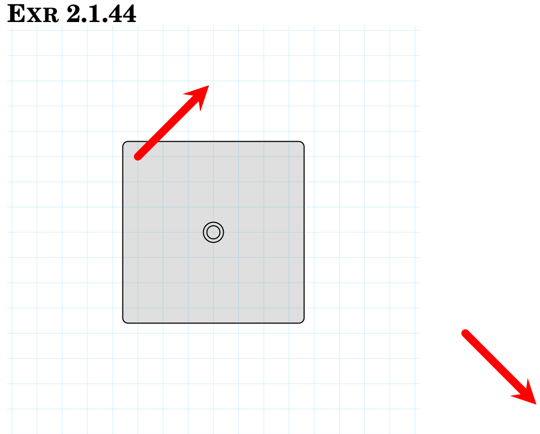
Exr 2.1.43



Exr 2.1.41

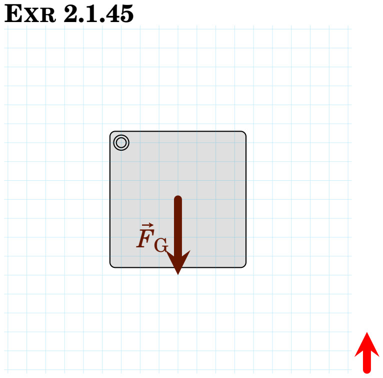


Exr 2.1.44

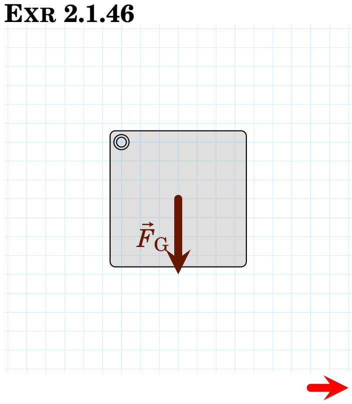


In this next set of exercises the pivot is ***not*** at the center of gravity of the object. This means that gravity will exert a torque about the pivot on the object. (The force of gravity $\vec{F}_{\mathrm{G}}$ is coloured dark brown in the diagrams.) The goal will be to find or place the force that will maintain equilibrium.

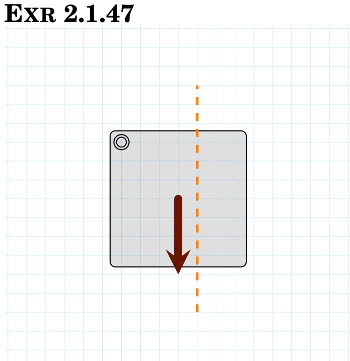
Exr 2.1.45



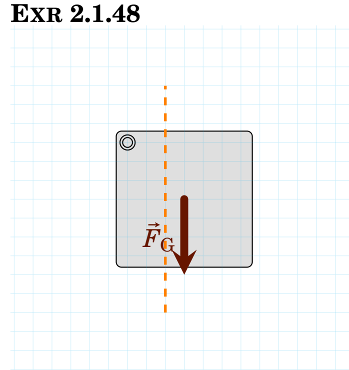
Exr 2.1.46



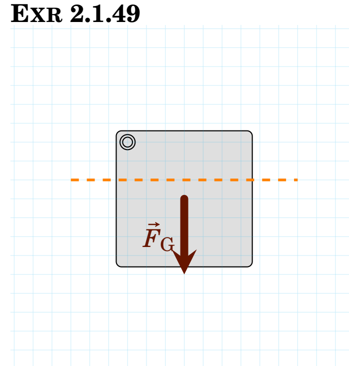
Exr 2.1.47



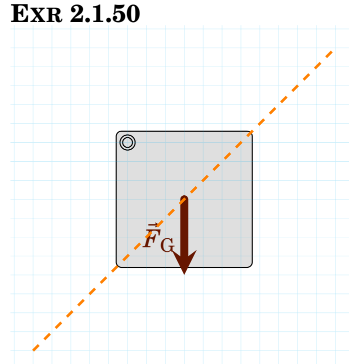
Exr 2.1.48



Exr 2.1.49

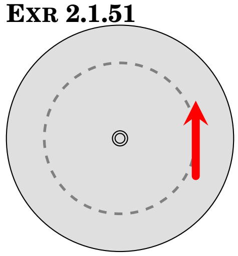


Exr 2.1.50

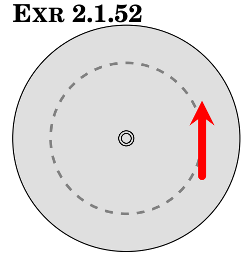


Circular objects

Exr 2.1.51



Exr 2.1.52



Irregularly-shaped objects

▲**FIX:** TO BE WRITTEN

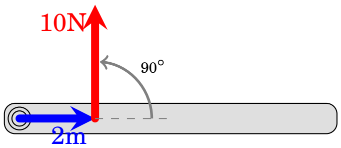
2.2 Torque : Quantitative Exercises

2.2.1 Single applied Force

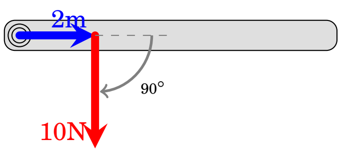
In each of these exercises determine:
(1) the angle θ from the direction of $\vec{r}$ to the direction of $\vec{F}$, and hence the sign of τ_z;
(2) the component $F_\perp$ of the force that exerts a torque; and
(3) the z-component of the torque (τ_z) exerted about the pivot by the applied force.

Right Angles

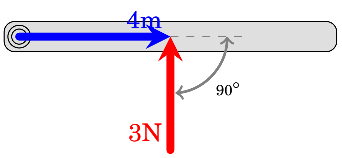
Exr 2.2.01



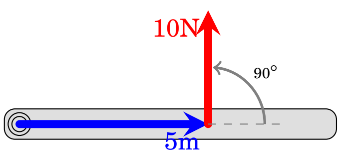
Exr 2.2.02



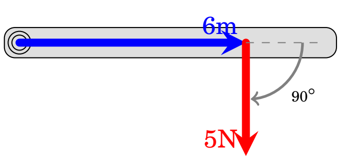
Exr 2.2.03



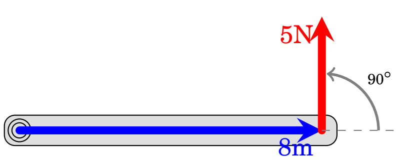
Exr 2.2.04



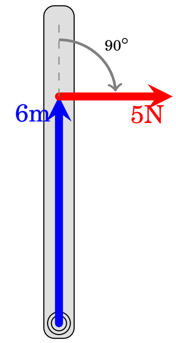
Exr 2.2.05



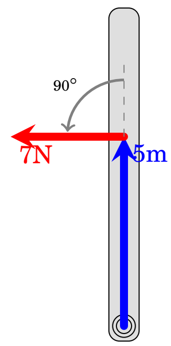
Exr 2.2.06



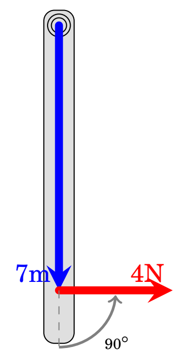
Exr 2.2.07



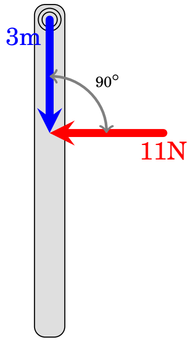
Exr 2.2.08



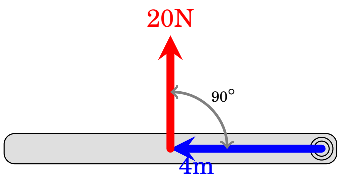
Exr 2.2.09



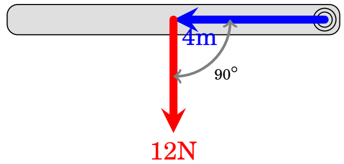
Exr 2.2.10



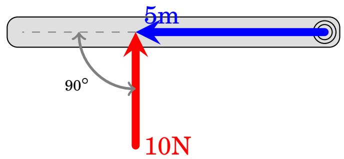
Exr 2.2.11



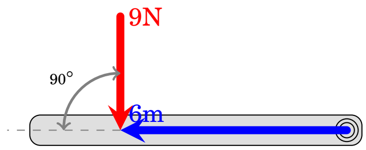
Exr 2.2.12



Exr 2.2.13

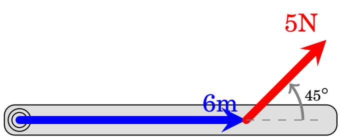


Exr 2.2.14

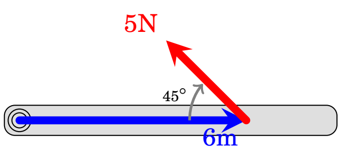


Non-Right Angles

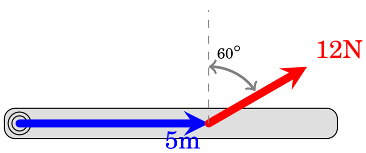
Exr 2.2.15



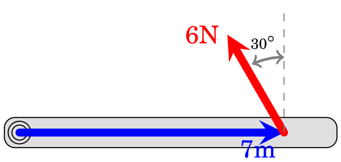
Exr 2.2.16



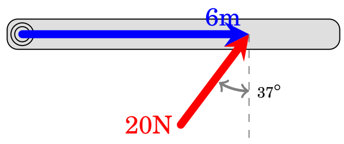
Exr 2.2.17



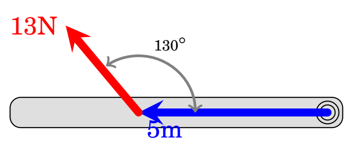
Exr 2.2.18



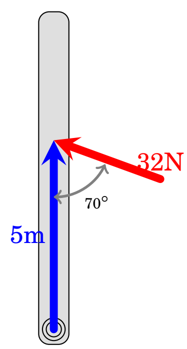
Exr 2.2.19



Exr 2.2.20



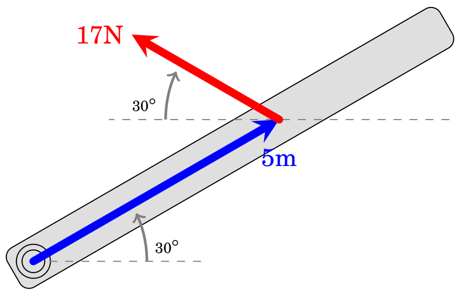
Exr 2.2.21



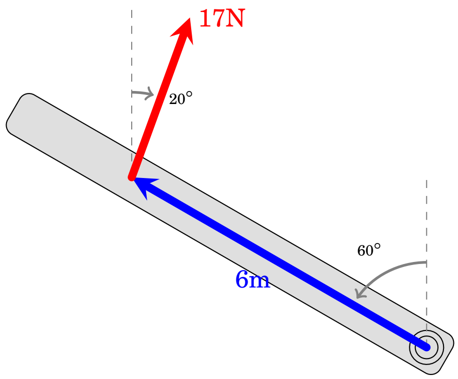
General Cases

In these cases you must solve the geometry to find the angle θ from the direction of $\vec{r}$ towards the direction of $\vec{F}$. Recall how to identify complementary angles, and rules like the transverse-parallel theorem. Be very careful to get the sign of θ correct. These exercises may require a few steps to solve the geometry. Be sure to draw larger diagrams than usual so that you can label the parts of the geometry clearly.

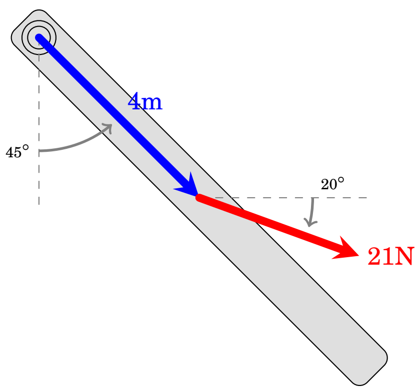
Exercise 2.2.22



Exercise 2.2.23



Exercise 2.2.24 (Challenge)



2.3 Solving Problems using *the Process*

The Process

0. Identify the Object!

1. Identify the forces acting on the Object.

2. Draw the Free-Body Diagram, clearly identifying the pivot.

3. Separately, for each force acting on the Object:

- draw the object and the coordinates
- draw the force, placing it on the object where it is acting
- draw the position vector $\vec{r}$ from the pivot to where the force is acting
- determine the components of the force, and the contribution to torque.

4. Use Newton's 1st Law to write the equations to be solved.

5. Solve the equations for the unknown quantities.

2.3.1 Solving for Forces using Torque

A mechanical system is said to be in static equilibrium when the sum of forces is zero and the sum of torques is also zero:

$$\sum F_x = 0\,\mathrm{N} \tag{2.1}$$
$$\sum F_y = 0\,\mathrm{N} \tag{2.2}$$
$$\sum \tau_z = 0\,\mathrm{N{\cdot}m} \tag{2.3}$$

In the exercises below we will begin by practicing how to solve for forces quantitatively using the equilibrium of torques. We will begin with simple geometric shapes, like squares, rectangles and circles. After that we will practice applying the rules of static equilibrium to mechanical objects of various shapes. Follow the steps of *The Process*!

Rectangular Solids

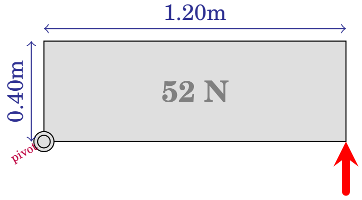
In each of the exercises that follow determine:

(1) Find the magnitude of the applied force that keeps the box in equilibrium; and

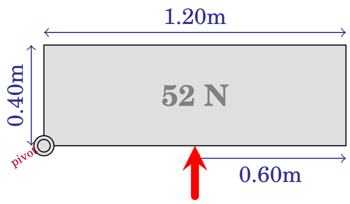
(2) Find the magnitude and direction of the force acting at the pivot.

In all cases, unless otherwise indicated, the force of gravity acts at the geometric center of the object. The magnitude of $\vec{F}_{\mathrm{G}}$ is the value printed on the object.

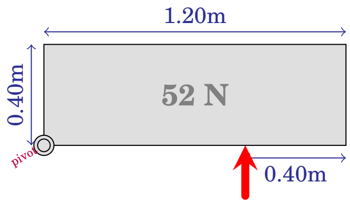
EXERCISE 2.3.01



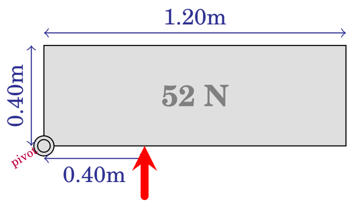
EXERCISE 2.3.02



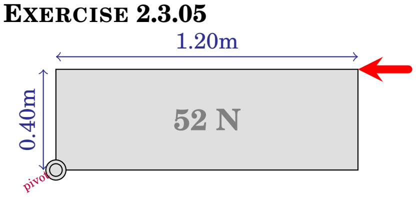
EXERCISE 2.3.03



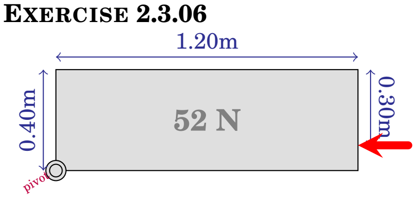
EXERCISE 2.3.04



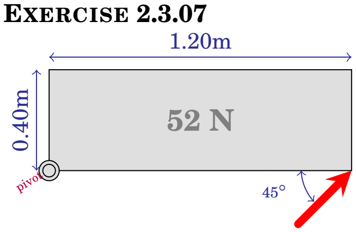
EXERCISE 2.3.05



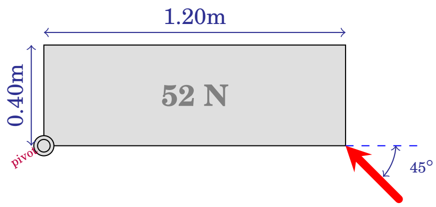
EXERCISE 2.3.06



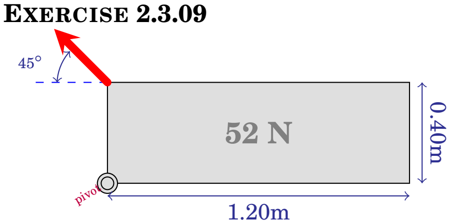
EXERCISE 2.3.07



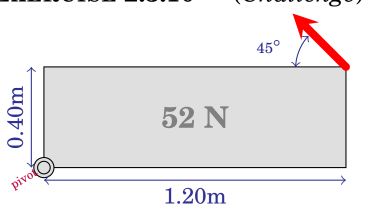
EXERCISE 2.3.08



EXERCISE 2.3.09



EXERCISE 2.3.10 (*Challenge*)

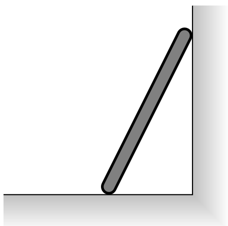


Rods, Ropes & Surfaces

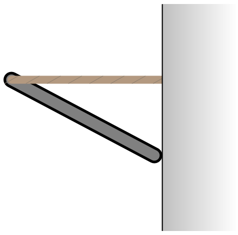
In each of the following situations, solve for each of the unknown forces. Find the normal at each surface of contact. In the cases when it is non-zero, find the magnitude and direction of friction. Find the tension in each rope that is present.

A hint on how to start problems of this type: Chose a point of contact with a surface as the pivot, preferably one where there is friction (when it is present). This will eliminate one (perhaps two!) unknowns from the torque equation, simplifying the algebra.

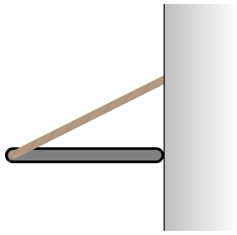
EXERCISE 2.3.11



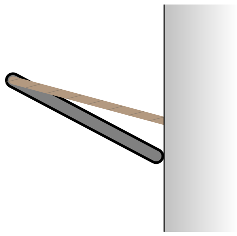
EXERCISE 2.3.12



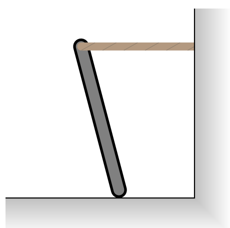
EXERCISE 2.3.13



EXERCISE 2.3.14



EXERCISE 2.3.15

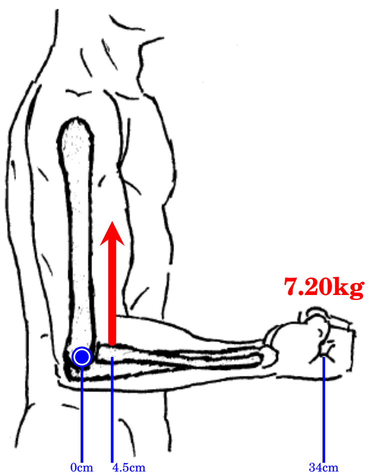


2.3.2 Biomechanical Equilibrium

In all the problems that follow, after finding the unknown force, find the force exerted at the pivot.

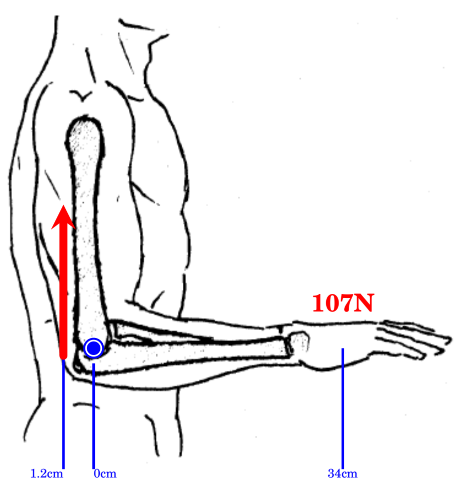
PROBLEM 2.3.01:

A person is holding a 7.20 kg mass in their hand. What total amount of force must the muscles of their upper arm be exerting on their lower arm? The pivot in this situation is the elbow joint. The active muscles are the biceps and brachialis, which are the large muscles on the front side of the upper arm. They attach to the forearm 4.5 cm from the elbow joint, as shown in the diagram. The forearm (including the hand) has a mass of 1.80 kg, and has its center of mass 15.0 cm from the elbow joint.



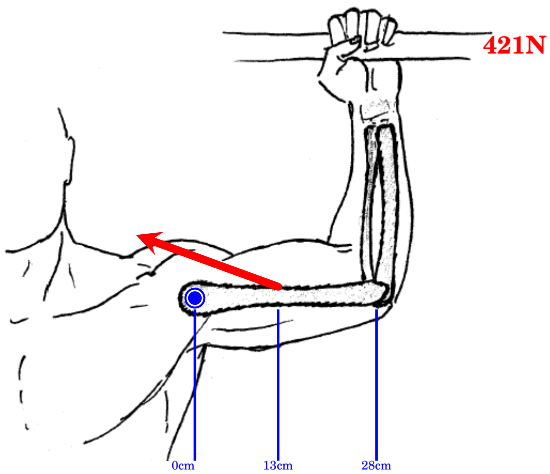
PROBLEM 2.3.02:

A person is pressing downwards with their hand, exerting 107 N on a surface (not shown). What total amount of force must the muscles of their upper arm be exerting on their lower arm? (The active muscles are the cluster referred to as the triceps, which are the muscles on the rear side of the upper arm.) The forearm (including the hand) has a mass of 1.8kg, and has its center of mass 15cm from the elbow joint.



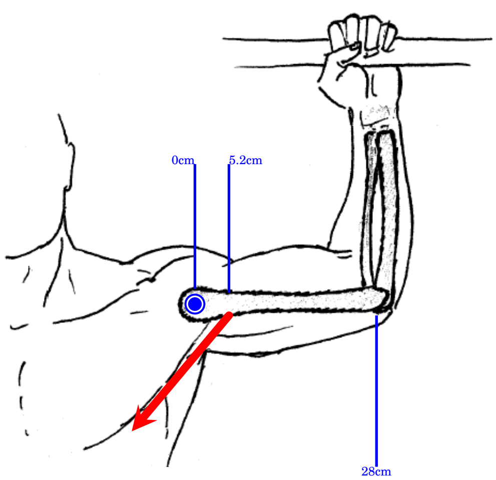
PROBLEM 2.3.03:

A weightlifter is holding a weight above their head. Each arm supports 421N. The muscles of the shoulder collectively exert a force along a line that is 20° above the axis of the bone of the upper arm (the humerus). What is the magnitude of that collective force? (We will neglect the mass of the arm itself in this.)



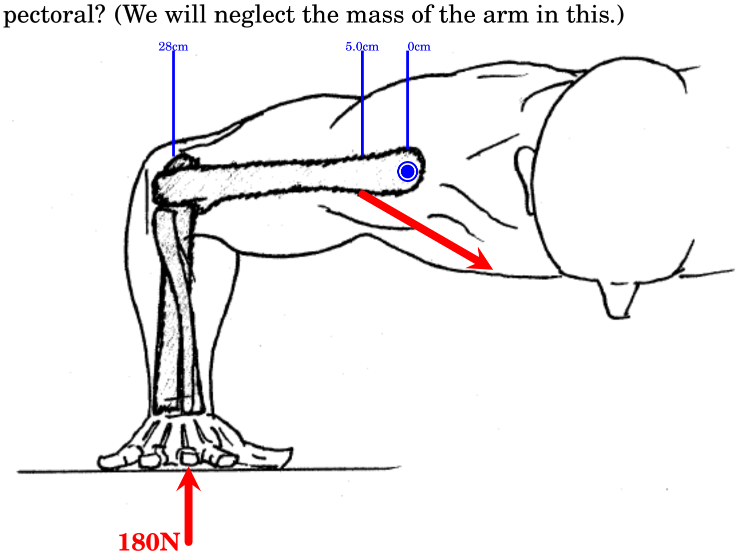
PROBLEM 2.3.04:

A person is doing a pull-up. Each arm support half their mass (total mass 70kg). The pectorals and the muscles of the back collectively exert a force along a line that is 40° to the left of the vertical. What is the magnitude of that collective force? (We will neglect the mass of the arm itself in this.)



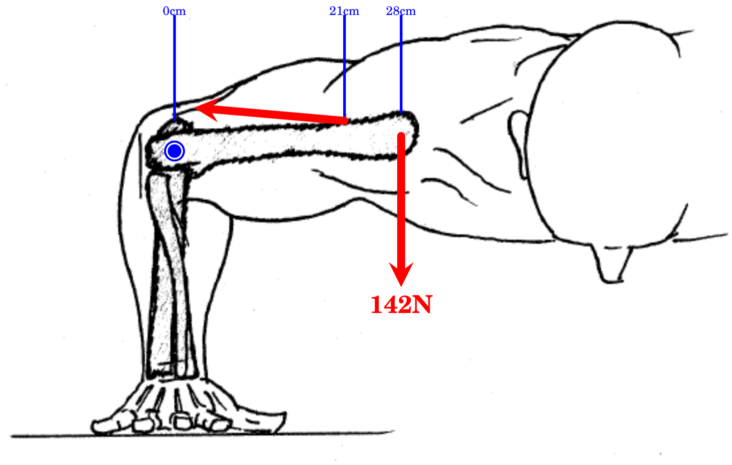
PROBLEM 2.3.05:

A person doing a push-up supports 180N at each hand. The pectoral muscle (the big muscle on the chest) attaches at an angle of 30° to the axis of the bone of the upper arm (the humerus). What is the magnitude of the force exerted by the pectoral? (We will neglect the mass of the arm in this.)



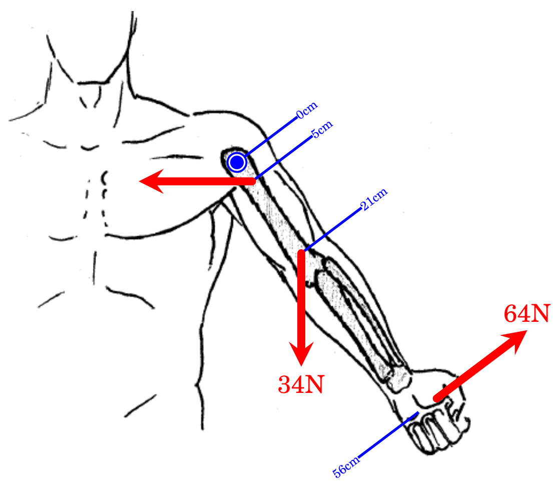
PROBLEM 2.3.06:

A person doing a push-up supports 142N at each shoulder joint. The triceps muscles (the muscles on the back of the upper arm) attaches at an angle of 5° to the axis of the bone of the upper arm (the humerus). What is the magnitude of the force exerted by the triceps? (We will neglect the mass of the arm in this.)



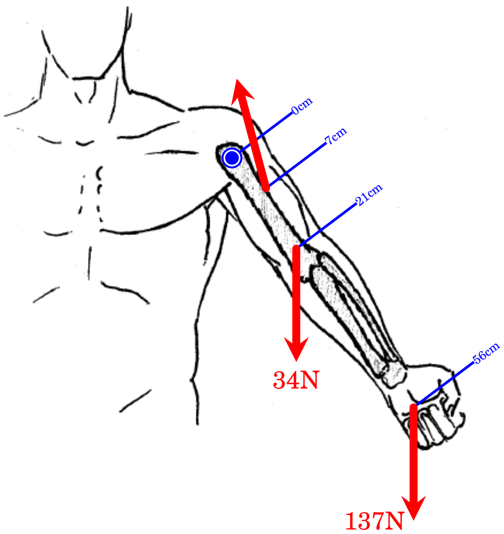
PROBLEM 2.3.07:

A person is holding a rope (not shown). The reaction force to their pull is the 64N force on their hand, perpendicular to their arm. The weight of their arm is 34N (center of mass 21cm from the shoulder). If their arm is 37° from the vertical, what force must their pectoral (chest muscle) exert horizontally?



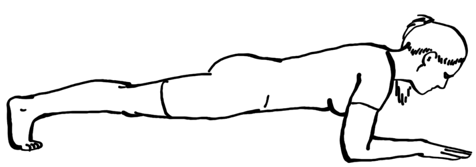
PROBLEM 2.3.08:

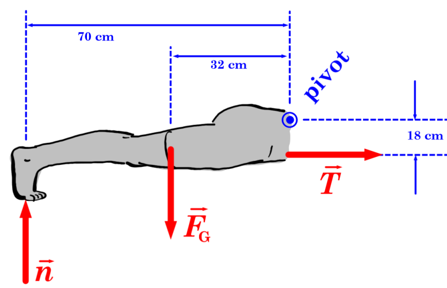
A person is lifting a 137N weight out to their side. The weight of their arm is 34N (center of mass 21cm from the shoulder). If their arm is 37° from the vertical, what force must their deltoid (shoulder muscle) exert? (The deltoid attaches to the humerus 7.0cm from the shoulder joint at an angle of 22°.)



PROBLEM 2.3.09:

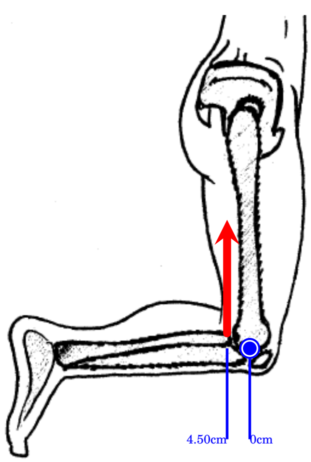
A woman doing core exercises is in "plank position", as shown in the diagram below. The joint where the spine meets the pelvis is the pivot, with the legs on one side and the upper body on the other. It is the abdominal muscles that keep the two segments from bending away from each other. The contact force at her feet is 230.0 N, and the mass of the lower segment of her body is 30.61 kg. What is the tension in her abdominal muscles that keeps her body in static equilibrium?





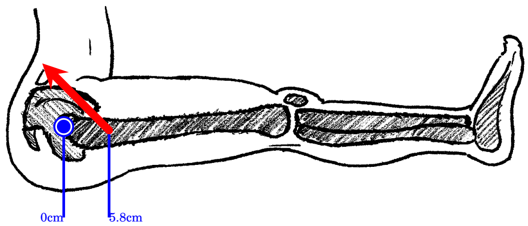
PROBLEM 2.3.10:

A person is standing vertically. Using their *hamstrings* (three muscles, of which the largest is the *biceps femoris*) they are holding the lower portion of one of their legs (3.00kg) horizontal, as shown. The muscles are attached 4.50cm from the pivot (the knee joint), and the center of mass of this segment is 15.3cm from the joint. What is the tension in the *hamstrings*?



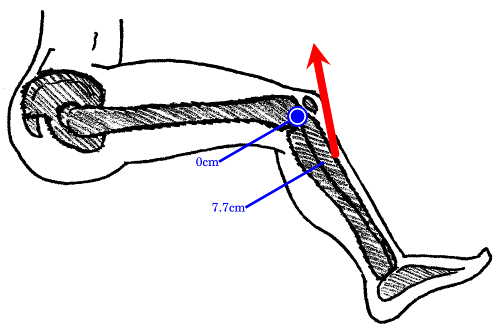
PROBLEM 2.3.11:

A vertical person is holding one of their legs horizontally, as shown. One of the major flexors, the *iliopsoas*, connects the *lesser trochanter* (a small bump on the femur) to portions of the pelvis and the lumbar region of the spine. This muscle exerts a net force pointed at 57° above the horizontal (as shown), acting 5.8 cm from the hip joint. The whole leg has a mass of 10.40 kg, and the center of mass is 29.2 cm from the hip joint. What is the tension in the muscle?



PROBLEM 2.3.12:

A vertical person with one of their legs as shown, is holding their lower leg at an angle of 30° from the vertical. This segment has a mass of 2.74 kg, with its center of mass 14.8 cm from the knee joint. The patellar ligament (the ligament that connects the patella [kneecap] to the tibia [shinbone]) attaches to the tibia at a point 7.7 cm from the joint, and the angle between the ligament and the tibia is 19°. What is the tension in the ligament?



Chapter 3

Materials

Remember that the symbol σ means "stress" (the way in which force is distributed across the cross-section of the object $\sigma = F/A$) and that the symbol ϵ means "strain" (the relative change in length due to the applied stress $\epsilon = \Delta L/L_\mathrm{i}$). Unless otherwise explicitly stated all the stresses are axial tension or axial compression.

In exercises involving circular geometry remember to find the *radius* in cases when the diameter is specified. When the cross-section is hollow, obtain the area for the stress by *subtracting* the area of the hollow from the area of the cross-section.

Since strain is the change in length divided by the original length ($\epsilon = \Delta L/L = (L_\mathrm{f} - L_\mathrm{i})/L_\mathrm{i}$), the new length of the deformed object is

$$L_\mathrm{f} = L_\mathrm{i} + \Delta L \tag{3.1}$$

$$L_\mathrm{f} = L_\mathrm{i} + \epsilon L_\mathrm{i} \tag{3.2}$$

Note also that since strain is a ratio of lengths, the units used do not matter, as long as they are the same.

In your calculations: be very careful with the units and powers of ten; keep at least four significant figures in your intermediate steps; and round only the final result.

3.1 Stress

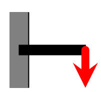
Exr 3.1.01 A 5.0 N force is applied across a rectangular surface measuring 2.0 cm by 7.0 cm. The stress is σ = ___ kPa.

Exr 3.1.02 A 1060 N force is applied across a rectangular surface measuring 10 cm by 30 cm. The stress is σ = __ kPa.

Exr 3.1.03 A 120 kg mass is resting on a rectangular surface measuring 8.0 cm by 27.0 cm. The pressure is P = __ kPa.

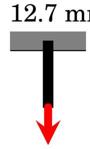
Exr 3.1.04 A force of 33.7 N is shearing across a rectangular surface measuring 5.0 mm by 8.0 mm. The shear stress is σ = ____ MPa.

Exr 3.1.05 A 7.16 kg mass is suspended at the end of a horizontal metal rod that is square in cross-section (width 1.00 cm). The *shear* stress is σ = _____ MPa.



Exr 3.1.06 A cylindrical rod of diameter 5.00 cm is under 12.0 N of compression. The stress is σ = ____ kPa.

Exr 3.1.07 A 621 kg mass hangs at the end of a 12.7 mm diameter metal rod. The stress is σ = ____ MPa.



Exr 3.1.08 A 7.00 kg mass hangs at the end of a cylindrical rod of diameter 6.0 mm. The stress is σ = ___ MPa.

Exr 3.1.09 A 7.16 kg mass is suspended at the end of a horizontal metal rod that is circular in cross-section (diameter 1.00 cm). The *shear* stress is σ = ____ MPa.

Exercise 3.1.10 A hollow square beam supports 12 507 N of weight. The outer width of the beam is 42.7 mm and the inner width of the hollow is 40.1 mm. The stress in the material of the beam is σ = ____ MPa.

Exercise 3.1.11 A 18.755 kg bowling ball rests on the end of a hollow cylinder with outer radius 3.30 cm and inner radius 3.20 cm. The stress in the material of the cylinder is σ = _____ MPa.

Exercise 3.1.12 A hollow metal tube with outer diameter 3.81 mm and inner diameter 3.77 mm is under a tension of 522 N. The stress in the material of the tube is σ = ____ GPa.

Exercise 3.1.13 A 1406 kg car rests on its four tires, each with a contact patch measuring 19 cm by 24 cm. The pressure on each tire is P = __ kPa.

Exr 3.1.14 A 54.4 kg ballerina stands on one foot (equivalent to a rectangle measuring 20.3 cm by 5.1 cm). The pressure under her foot is P = __ kPa.

Exr 3.1.15 A 54.4 kg ballerina stands on the point of her toes of one foot (equivalent to a rectangle measuring 6.0 cm by 2.0 cm). The pressure under her toes is P = ____ MPa.

Exr 3.1.16 An adult male moose can have masses up to 700 kg! Their femur is a hollow cylinder of bone, outer diameter 5.5 cm, inner diameter 3.5 cm. The stress in the bone is σ = ___ MPa. (Assume that its weight is distributed equally onto each of their four legs.)

3.2 Strain

While doing these exercises and problems remember:

- Strain is defined to be a ratio of lengths, so make sure that both quantities have the same units.
- Do not round any of the values you are using until you reach the final result. The change in length will be the subtraction of two numbers that (typically) differ only in their last few decimal places; do not truncate any of those.
- The sign of the change in length, and hence the sign of the strain, is *very* important. Double-check that it is correct: $\epsilon > 0$ means an increase in length, while $\epsilon < 0$ means a decrease in length. In your answer be explicit about the sign.
- With the exception of extremely soft elastic materials (like rubber) realistic strains should be very small numbers. If you find a strain $\epsilon > 1$, then you've probably made a mistake somewhere.

Exr 3.2.01 A meter stick (length 1.000 m) is subjected to tension and stretches to 1.003 m. The strain is ϵ = ________.

Exr 3.2.02 An elastic band is stretched from 8.5 cm to 14.2 cm. The strain is ϵ = _____.

Exr 3.2.03 An object of length 45.00 cm is subjected to a stress and compresses to 44.92 cm. The strain is ϵ = ________.

Exr 3.2.04 A steel ruler (length 12.02 in) is subjected to tension and stretches to 12.03 in. The strain is ϵ = _____%. (We are looking for the strain expressed as a percent.)

Exr 3.2.05 A wooden column of height 67.02 inches is subjected to a stress and compresses to 66.97 inches. The strain is ϵ = ________.

Exr 3.2.06 A ruler (original length 30.22 cm) is subjected to tension and exhibits a strain of $\epsilon = +0.20\%$. The deformed length is L_f = _____ cm.

Exr 3.2.07 A segment of a bridge (original length 27.052 m) is subjected to a stress and exhibits a strain of $\epsilon = +3.5 \times 10^{-5}$. The deformed length is L_f = ______ m.

Exr 3.2.08 A segment of a bridge (original length 27.052 m) is subjected to a stress and exhibits a strain of $\epsilon = -3.5 \times 10^{-5}$. The deformed length is L_f = ______ m.

Exr 3.2.09 When the ground under a building subsides a concrete wall (original height 13.09 m) is subjected to a stress and exhibits a strain of $\epsilon = +7.30 \times 10^{-6}$. The deformed height is L_f = _____ m.

Exr 3.2.10 A person out on a nature hike takes a rest on a granite boulder (original height 75.00 cm). The compressive stress due to the person's weight causes a strain of $\epsilon = -5.28 \times 10^{-8}$ in the boulder. The deformed height is L_f = _____ cm.

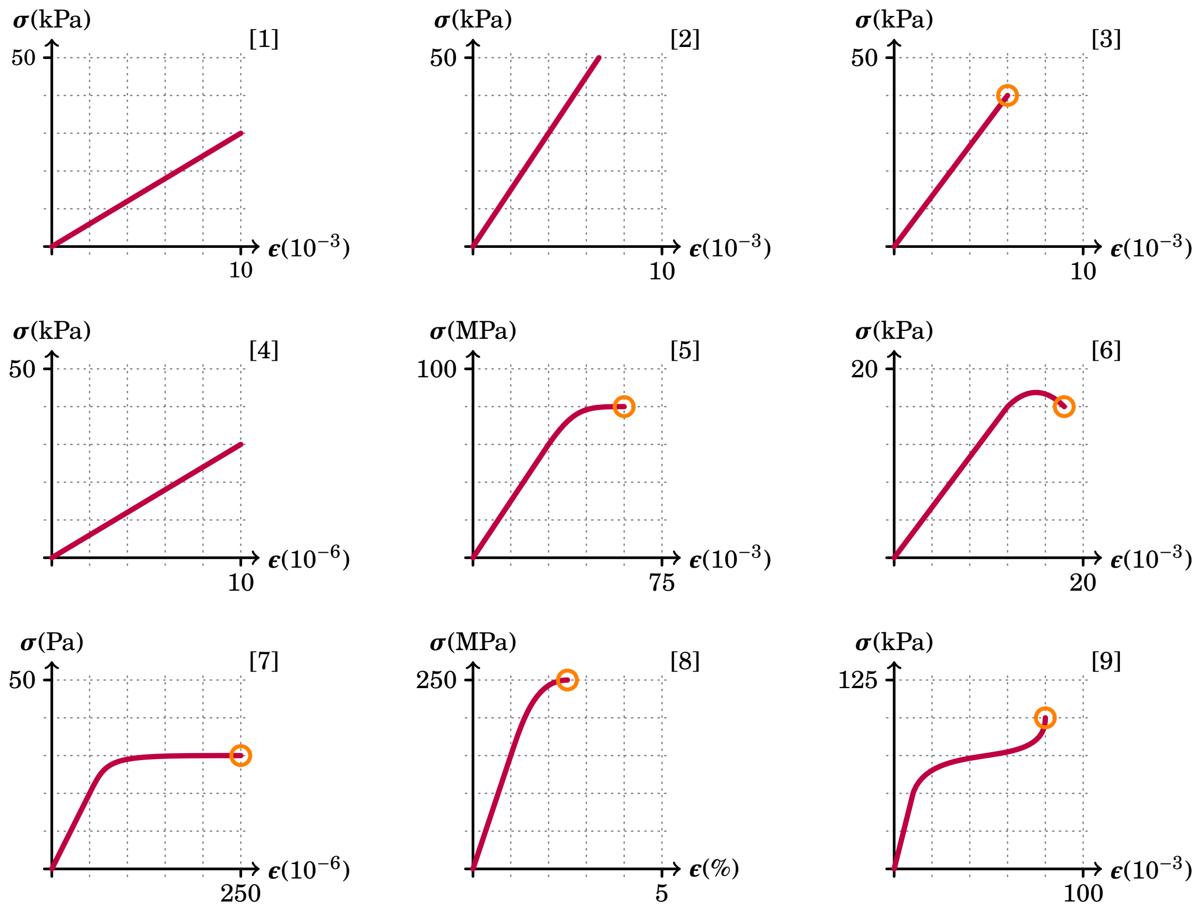
3.3 Stress-Strain Curves

In each of the stress-strain graphs of the following exercises a small orange circle around the end of a stress-strain curve indicates the failure of the material.

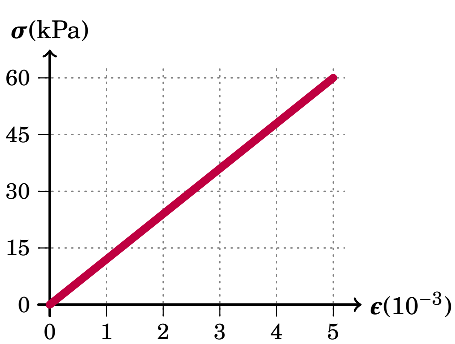
EXERCISE 3.3.01 Consider the stress-strain curves plotted below.

(a) Calculate the Young's Modulus for each of the materials. (Be careful with the units!)

(b) For those materials that have a failure in the plotted range of strains, estimate the stress and strain at failure. For those graphs also estimate their values at the yield point (if it exists).

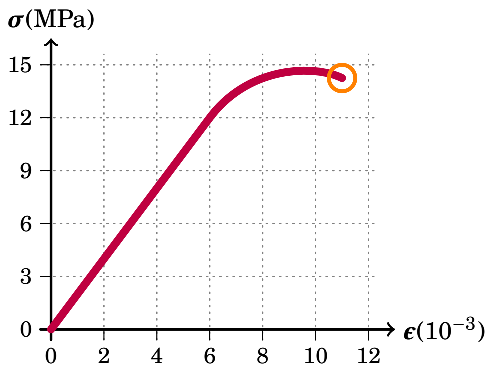


PROBLEM 3.3.01:



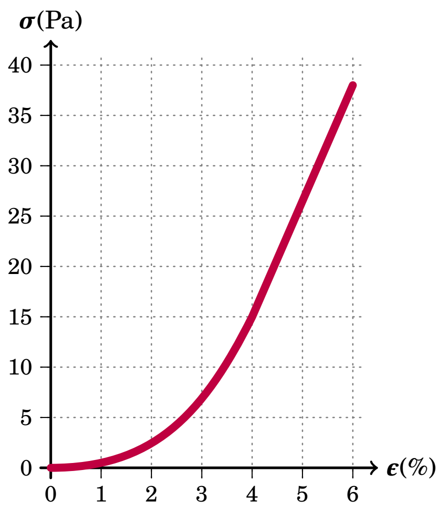
Given the stress-strain curve on the right:
(a) What is the Young's modulus of the material?
(b) What is stress in the material when the strain is $\epsilon = 3.5 \times 10^{-3}$?
(c) What is strain in the material when the stress is $\sigma = 20\,\text{kPa}$?

PROBLEM 3.3.02:



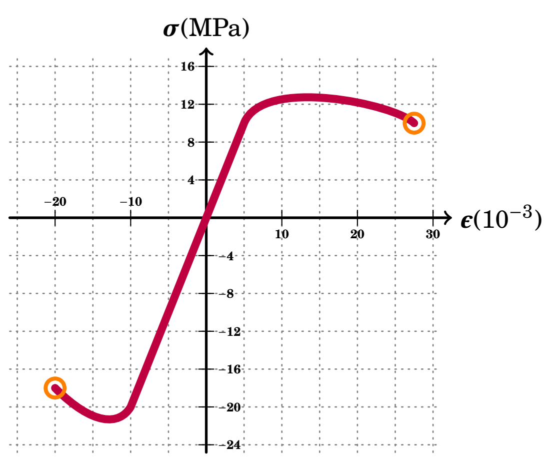
Given the stress-strain curve on the right:
(a) What is the strain at the yield point? If the object were 14.0 cm in length when unstressed, what would be its length at this limit?
(b) What is the Young's modulus of the material in the elastic regime?
(c) What is stress in the material when the strain is $\epsilon = 3.0 \times 10^{-3}$?
(d) What is the strain at failure?

PROBLEM 3.3.03:



Given the stress-strain curve on the right:
(a) From the shape of the stress-strain curve, what *type* of material is this?
(b) Estimate the strain of the material when the stress is $\sigma = 10\,\text{Pa}$.
(c) Estimate the value of Young's modulus in the regime where the stress-strain curve is linear.

PROBLEM 3.3.04:



The stress-strain curve above shows the behavior of the material both under compression and under tension. Given that curve:

(a) Estimate the stress and strain at yield when the material is under tension.

(b) Estimate the stress and strain at yield when the material is under compression.

(c) Estimate the value of strain at which the stress has its largest magnitude. Is that when the material is under tension or compression?

(d) Estimate the stress and strain when the material fails under tension.

(e) Estimate the value of Young's modulus for values of strain between the compressive yield point and the tensile yield point.

Chapter 4
Energy

Introduction.

4.1 Thermal Energy

4.1.1 Units

Exr 4.1.01 What is the temperature 300. K measured in Celsius?

Exr 4.1.02 The SI system is based on the metre, kilogram, and second. In this system the unit of energy, the joule, has units $kg \cdot m^2/s^2$. There is a related system of units: the *CGS* system, which is based on the centimetre, gram, and second. In that system the unit of energy is the ***erg***, which is $1\,erg = 1\,g \cdot cm^2/s^2$. How many ergs are in one joule?

Exr 4.1.03 The density of Earth's atmosphere (at 101.3 kPa and 15 °C) is approximately $1.225\,kg/m^3$. What is that density measured in grams per litre?

4.1.2 Thermal Energy and Temperature

Energy Transferred

Category 1: Using $\Delta E = m\mathscr{C}\,\Delta T$ to find the thermal energy ΔE transferred.

Exr 4.1.04 How many kilojoules of thermal energy must be added to increase the temperature of 3.00 L of water by 2.00 C°?

Exr 4.1.05 How many kilojoules of thermal energy must be removed from 650 mL of water to decrease its temperature by 16.00 C°?

Exr 4.1.06 A warm bathtub (172 L) cools by 0.25 C°: how much thermal energy does it lose?

Exr 4.1.07 To make a cup of tea we raise 0.53 L by 81 C°: how much thermal energy does this take, in kilojoules?

Temperature Change

Category 2: Using $\Delta E = m\mathscr{C}\,\Delta T$ to find the change $\Delta T = T_{\text{final}} - T_{\text{initial}}$ in temperature.

Exr 4.1.08 By how much does the temperature of 3.00 L of water change if we transfer 50.2 kJ into it?

Exr 4.1.09 By how much does the temperature of 650 mL of water change if we transfer 54.4 kJ out of it?

Exr 4.1.10 By how much does the temperature of 3.00 L of water change if we transfer 753 kJ into it?

Exr 4.1.11 By how much does the temperature of 650 mL of water change if we transfer 109 kJ out of it?

Temperature, initial or final

Category 3: Using $\Delta E = m\mathscr{C}\,\Delta T$ to find either the initial temperature T_{initial}, or the final temperature T_{final} from $\Delta T = T_{\text{final}} - T_{\text{initial}}$.

Exr 4.1.12 We add 50.2 kJ of thermal energy to 3.00 L of water that begins at +18 °C. What is its final temperature?

Exr 4.1.13 730 mL of hot water is allowed to cool. After it has lost 125 kJ of thermal energy the water has cooled to a temperature of +23.1 °C. How hot was it to begin? (What was its initial temperature?)

Exr 4.1.14 We remove 301 kJ of thermal energy from 1.80 L of water that begins at +100. °C (it has just finished boiling). What is its final temperature?

Exr 4.1.15 12.5 L of warm water was being heated. After it has gained 1.674 MJ of thermal energy the water has reached a temperature of +92.0 °C. How warm was it to begin? (What was its initial temperature?)

Equilibrium Temperature

Category 4: Mixing together two amounts of water (m_A and m_B) at different initial temperatures (T_{Ai} and T_{Bi}), the thermal energy that one gains will equal the thermal energy lost by the other. This is conservation of energy: $\Delta E_A + \Delta E_B = 0$. Once the system has reached equilibrium the combined mass of water will be at a common final temperature. From conservation of energy we can solve for unknown temperatures or masses:

$$T_{\text{final}} = \frac{m_A T_{Ai} + m_B T_{Bi}}{m_A + m_B} \tag{4.1}$$

(You can practice your algebra skills by deriving this equation, if you want to.)

Exercise 4.1.16 We mix together 1.000 L of water at +30 °C with 1.000 L of water at +20 °C. What will be the final temperature of this mixture?

Exercise 4.1.17 We mix together 800 mL of water at +30 °C with 1.200 L of water at +20 °C. What will be the final temperature of this mixture?

Exercise 4.1.18 We mix together 1.200 L of water at +30 °C with 800 mL of water at +20 °C. What will be the final temperature of this mixture?

Exercise 4.1.19 We mix together 1.000 L of water at +30 °C with 93 mL of water that was just boiled +100 °C. What will be the final temperature of this mixture?

4.1.3 Power

The relation between power, time and energy is

$$\Delta E = P \times \Delta t \tag{4.2}$$

In the context of thermal energy and its relation to temperature, this means that

$$m\mathscr{C}\,\Delta T = P \times \Delta t \tag{4.3}$$

Here we must be careful to not confuse the change in temperature ΔT and the interval of time Δt.

The rate at which thermal energy is transferred by conduction through a boundary of area A and thickness L is

$$P = \mathscr{K}\,\frac{A}{L}\left(T_{\text{env}} - T_{\text{sys}}\right) \tag{4.4}$$

where $T_{\text{env}} - T_{\text{sys}}$ is the temperature difference across the boundary (between the system inside the boundary, and the environment outside the boundary). In that expression $\mathscr{K}$ is the *thermal conductivity* of boundary's material, which measures how easily thermal energy is transferred through the material.

Exr 4.1.20 If the temperature of 1 L of water is changing at rate of +1.37 C° every 5.2 s, at what rate is thermal energy being transferred to the water?

Exr 4.1.21 If a 1.200 kW microwave runs for 60 s by what increment would the temperature of a 250 mL cup of water increase?

Exr 4.1.22 If a 118 mL cup of tea is cooling at a rate of 23 W, how long will it take to cool 3 C°? (Assume that the tea has the same heat capacity as water.)

4.1.4 Problems

Problem 4.1.01: A large volume of water at 23.2 °C is separated from a large volume of water at 45.8 °C by a steel wall of area 0.0223 m^2 and thickness 3.07 mm. At these temperatures the thermal conductivity of steel is approximately

$\mathscr{K} \approx 13.5\,\mathrm{W/m\cdot K}$.
(a) At what rate is heat flowing through the steel separator?
(b) If we started with identical initial conditions but the steel separator was replaced with a glass separator ($\mathscr{K} \approx 0.96\,\mathrm{W/m\cdot K}$) of identical size and thickness, what would be the rate?

PROBLEM 4.1.02: A person, dressed in winter clothes, is standing outside in February in Montreal. Their clothes (and body fat!) have a combined thermal conductivity of $\mathscr{K} = 0.047\,\mathrm{W/m\cdot K}$, and an effective thickness of 37 mm. The air temperature is $-25\,°\mathrm{C}$, and their internal body temperature is $+37\,°\mathrm{C}$. Assuming a surface area of approximately $1.7\,\mathrm{m}^2$, at what rate are they losing thermal energy?

PROBLEM 4.1.03: How long would it take a 500 W heater to increase the air temperature from 17 °C to 21 °C in room that measures 5.05 m by 4.33 m and is 2.25 m tall? (Air has a density $\rho = 1.225\,\mathrm{kg/m^3}$, and its heat capacity is $\mathscr{C}_{\mathrm{air}} = 1.006\,\mathrm{kJ/kg\cdot C°}$.)

PROBLEM 4.1.04: A 6.117 kg block of steel is placed into a 14.3 L insulated container of water, and then sealed. If the water was initially at 19.5 °C and the steel was at 95.8 °C, what will be their final equilibrium temperature? (Use $\mathscr{C}_{\mathrm{steel}} = 497\,\mathrm{J/kg\cdot K}$, and assume that very little thermal energy is exchanged with the surroundings.)

PROBLEM 4.1.05: In room that measures 5.05 m by 4.33 m and is 2.25 m tall? the air temperature is 17.0 °C. If a 2.000 L container of water at 42.0 °C is placed in this room, what would be the equilibrium temperature of the water and the air in the room? (Ignore any transfer of thermal energy with anything else, like the walls, or anything outside; this is just a crude estimate. Use the facts that air has a density $\rho = 1.225\,\mathrm{kg/m^3}$, and its heat capacity is $\mathscr{C}_{\mathrm{air}} = 1.006\,\mathrm{kJ/kg\cdot C°}$.)

4.2 Mechanical Energy

4.2.1 Kinetic Energy

The linear kinetic energy of an object of mass m moving with speed v is

$$K_{\text{lin.}} = \tfrac{1}{2} m v^2 \tag{4.5}$$

When the mass is measured in kilograms and the speed is measured in metres per second, the kinetic energy calculated by this formula is in joules. The linear (or *translational*) speed can be calculated by

$$v = \Delta x / \Delta t \tag{4.6}$$

where Δx is the change in the object's position (measured along a straight line), and Δt is the time taken to travel that distance.

The angular kinetic energy of an object with moment of inertia $\mathscr{I}$ rotating with angular speed ω is

$$K_{\text{ang.}} = \tfrac{1}{2} \mathscr{I} \omega^2 \tag{4.7}$$

When the moment of inertia is measured in kilograms times metres-squared and the angular speed is measured in radians per second, the kinetic energy calculated by this formula is in joules. The angular (or *rotational*) speed can be calculated by

$$\omega = \Delta\theta / \Delta t \tag{4.8}$$

where $\Delta\theta$ is the change in the object's orientation (measured by a change in angle), and Δt is the time taken to travel that distance. The angular change must be measured in ***radians***:

$$2\pi\,\text{rad} = 360^\circ \tag{4.9}$$

Another common unit of rotation is the measure of the number of *revolutions* of the object (where $1\,\text{rev} = 2\pi\,\text{rad}$), and the speed in revolutions per second, or revolutions per minute (rpm):

$$1\,\text{rps} = \frac{1\,\text{rev}}{1\,\text{s}} = \frac{2\pi\,\text{rad}}{1\,\text{s}} \approx 6.23\,\text{rad/s} \tag{4.10}$$

$$1\,\text{rpm} = \frac{1\,\text{rev}}{1\,\text{min}} = \frac{2\pi\,\text{rad}}{60\,\text{s}} \approx 0.1047\,\text{rad/s} \tag{4.11}$$

If an object (or system of objects) has parts moving independently of each other, then the kinetic energy of the system is just the sum of the kinetic energies of the parts. If an object is rotating while it is also moving from place to place, then its kinetic energy is the sum of its linear kinetic energy and its angular kinetic energy.

Linear Kinetic Energy

Exr 4.2.01 What is the kinetic energy of a baseball (mass 153 grams) thrown at a speed of 23 m/s?

Exr 4.2.02 What is the kinetic energy of a bird (mass 74.8 grams) flying at a speed of 11.6 m/s?

Exr 4.2.03 What is the kinetic energy of a bowling ball (mass 5.17 kg) moving at 2.21 m/s?

Exr 4.2.04 What is the kinetic energy of a car (mass 1341 kg) driving at a speed of 10.0 km/h?

Exr 4.2.05 What is the kinetic energy of a car (mass 1341 kg) driving at a speed of 20.0 km/h?

Exr 4.2.06 What is the kinetic energy of a car (mass 1341 kg) driving at a speed of 100.0 km/h?

Exr 4.2.07 At what speed (in km/h) is the kinetic energy of a 1200 kg car equal to the energy (approximately 1 million joules) released by the explosion of one stick of TNT?

Exr 4.2.08 What is the kinetic energy of a person (mass 65.2 kg) walking at a speed of 6.44 km/h?

Exr 4.2.09 What was the kinetic energy of Usain Bolt (mass 94 kg) sprinting at a speed of 10.0 m/s?

Exr 4.2.10 A group of twenty-seven young children are running around, fueled by Halloween candy. In this group the average mass and speed of a child are 37.1 kg and 3.87 m/s. What is the kinetic energy present in this group of children?

Angular Kinetic Energy

Exr 4.2.11 An object with moment of inertia $\mathscr{I} = 0.875\,\mathrm{kg\cdot m^2}$ is rotating at $4.00\,\mathrm{rad/s}$. What is its angular kinetic energy?

Exr 4.2.12 An object spinning rapidly at $37.1\,\mathrm{rad/s}$ has $1.28\,\mathrm{J}$ of angular kinetic energy. What is its moment of inertia?

Exr 4.2.13 An object is rotating at $45.00\,\mathrm{rpm}$ (revolutions per minute). If its moment of inertia is $\mathscr{I} = 1.333\,\mathrm{kg\cdot m^2}$ what is its rotational kinetic energy?

Exr 4.2.14 An object with moment of inertia $\mathscr{I} = 0.411\,\mathrm{kg\cdot m^2}$ turns half-way around (180°) in $0.604\,\mathrm{s}$. What is its rotational kinetic energy?

Exr 4.2.15 A hollow sphere with moment of inertia $\mathscr{I} = 0.03820\,\mathrm{kg\cdot m^2}$ completes 7 rotations every 12 seconds. What is its angular kinetic energy?

Exr 4.2.16 A rod, pivoted about its end, with moment of inertia $\mathscr{I} = 0.420\,\mathrm{kg\cdot m^2}$ swings 37° in one third of a second. What is its angular kinetic energy?

The next few exercises are considered *advanced* in that they require you to calculate the moment to inertia from a formula particular to the object's geometry (*which I will give to you*).

Exercise 4.2.17 A BluRay disk (diameter 12 cm, mass 0.067 kg) is rotating at 810 rev/s. What is its angular kinetic energy? (Ignoring the hole in the middle, the moment of inertia is given by $\mathscr{I} = \frac{1}{2}mR^2$ approximately.)

Exercise 4.2.18 A basketball is spinning at three revolutions per second. The basketball is a hollow sphere of circumference 74.9 cm and mass 0.624 kg. What is its angular kinetic energy? (The moment of inertia of a hollow sphere, about an axis through its center, is $\mathscr{I} = \frac{2}{3}mR^2$. Be careful finding the radius!)

Exercise 4.2.19 A rectangular rod (mass 5.18 kg, length 41.1 cm, width 17.0 cm) swings about its end. It swings 25° in 0.25 s. What is its angular kinetic energy? (A rectangular rod of length ℓ and width w has a moment of inertia about an axis at its end given by $\mathscr{I} = m\left(\frac{1}{12}w^2 + \frac{1}{3}\ell^2\right)$.)

4.2.2 Work

When a force $\vec{F}$ is applied to an object, if the object moves a distance $\vec{r}$ along a straight line (its *displacement*), then the ***work*** done to the object is

$$W = F\,r\cos\theta \tag{4.12}$$

where θ is the angle between the force $\vec{F}$ and the displacement $\vec{r}$. This transfer of energy (either into the object or out of the object) changes the kinetic energy of the object

$$\Delta K = W \tag{4.13}$$

where $\Delta K = K_{\text{final}} - K_{\text{initial}}$, and $K = \frac{1}{2}m\,v^2$ is the linear kinetic energy.

When a torque $\vec{\tau}$ is applied to an object, if the object turns through an angle $\Delta\theta$ (its *angular displacement*) about the axis defined by the direction of $\vec{\tau}$, then the work done to the object is

$$W = \tau_z\,\Delta\theta \tag{4.14}$$

This transfer of energy (either into the object or out of the object) changes the kinetic energy of the object

$$\Delta K = W \tag{4.15}$$

where $\Delta K = K_{\text{final}} - K_{\text{initial}}$, and $K = \frac{1}{2}\mathscr{I}\,\omega^2$ is the angular kinetic energy.

EXERCISE 4.2.20 What is the work done by a 5.00 N force applied across 70.0 cm if the force is parallel to the displacement?

EXERCISE 4.2.21 What is the work done by a 1.11 N force applied across 1.23 m if the force points opposite the displacement?

EXERCISE 4.2.22 What is the work done by a 32.00 N force applied across 2.75 m if the force points 30° above the direction of the displacement?

EXERCISE 4.2.23 What is the work done by a 42.42 N force applied across 5.05 m if the force points 69° above the direction *opposite* the displacement? (Geometrically, if the object is moving along the $+x$-axis, the force points 69° above the $-x$-axis.)

EXERCISE 4.2.24 What amount of work needs to be done to a fully loaded shopping cart (39.3 kg) to reduce its speed from 1.72 m/s to 0.23 m/s?

4.2.3 Potential Energy

"Potential energy" is the term used to describe forms of energy that are not kinetic energy, but that may (through interaction) become kinetic energy.

$$U_{\text{G}} = mg\,h \tag{4.16}$$

where h is the vertical distance above ($h > 0\,\text{m}$) or below ($h < 0\,\text{m}$) the position *chosen* to be the "zero" gravitational potential.

When a material has its size deformed by an amount ΔL, if the deformation is in its elastic regime the energy stored in the material is given by

$$U_{\text{elastic}} = \frac{1}{2}k\,(\Delta L)^2 \tag{4.17}$$

It is important to note that ΔL must measure the deformation from the material's equilibrium (unstressed) size, *not* from some arbitrary "initial" size.

EXR 4.2.25 If a 0.375 kg object falls a distance 22.2 cm, by what amount does its gravitational potential change?

EXR 4.2.26 Go up. Go up a bit more. Then come down a little. Only $h_{\text{f}} - h_{\text{i}}$. The positions along the way do not matter.

EXR 4.2.27 Stretch a spring.

EXR 4.2.28 Stretch a spring that is already stretched.

4.2.4 Conservation of Energy

The fundamental principle of the Conservation of Energy is that if a collection of objects are interacting in ways that exchange (or transform) energy *only between themselves*, then the total energy of that ***system*** of objects does not change:

$$E_{\text{final}} = E_{\text{initial}} \tag{4.18}$$

This is also written

$$\Delta E_{\text{sys}} = 0\,\text{J} \tag{4.19}$$

When the total energy in the system is written as $E_{\text{sys}} = K + U + E_{\text{Th}}$ (where E_{Th} is the thermal energy generated in the system), conservation of energy can be written as

$$\Delta K + \Delta U + \Delta E_{\text{Th}} = 0\,\text{J} \tag{4.20}$$

4.2.5 Power & Efficiency

Power is the rate with respect to time of energy being transferred or transformed:

$$P = \Delta E / \Delta t \tag{4.21}$$

The units of power are *watts*: $1\,\text{W} = 1\,\text{J/s}$. Another unit of power that is sometimes used in the context of vehicles and machinery is the *horsepower*:

$$1\,\text{hp} = 746\,\text{W} \tag{4.22}$$

If we know the power and the amount of time, then we have the amount of energy that was transferred or transformed:

$$\Delta E = P \times \Delta t \tag{4.23}$$

In the context of electricity, where power is measured in kilowatts and time is measured in hours, the amount of energy is measured in *kilowatt-hours*

$$1\,\text{kWh} = 1\,\text{kW} \times 1\,\text{h} = 1\,000\,\text{W} \times 3\,600\,\text{s} = 3.6\,\text{MJ} \tag{4.24}$$

as an exact number. In the context of food energy, the *food calorie* (or kilocalorie) is meaningful: $1\,\text{Cal} = 4.184\,\text{kJ}$.

Efficiency measures the portion of energy that achieves the purpose. If an amount E_{input} is put towards a task, but only E_{output} is performed in the task then the efficiency is defined to be

$$E_{\text{output}} = \mathscr{E} \times E_{\text{input}} \tag{4.25}$$

For example, when gasoline is burnt in a car's engine to produce motion, only about 30% of the energy produced by the combustion manifests as kinetic energy of the car ($\mathscr{E} \approx 0.30$).

EXERCISE 4.2.29 If an old 60 W incandescent light bulb is left on for one hour, how much energy (in kilojoules) does it dissipate?

EXERCISE 4.2.30 If a 15 W LED light bulb is left on for one hour, how much energy (in kilojoules) does it dissipate?

EXERCISE 4.2.31 A 1200 W microwave is set to run at full power for 77 s. How much energy (in kilojoules) did it use?

EXERCISE 4.2.32 A television is left in its "stand-by" mode for eighteen hours. If that mode uses 24.0 W how much energy (in megajoules) did it use?

EXERCISE 4.2.33 Hydro Quebec charges 5.91 cents per kilowatt-hour of energy used. If an old 60W incandescent light bulb is left on for one whole week, how much will it cost?

4.2.6 Problems

PROBLEM 4.2.01: A 75.00 kg object moving at 5.165 $\frac{\text{m}}{\text{s}}$ has $-400\,\text{J}$ of work done to it.
(a) What is its initial kinetic energy?
(b) What will be its final kinetic energy?
(c) What will be its final speed? (Notice that the work done to the object is negative (we've removed energy from it) so its speed will decrease.)

PROBLEM 4.2.02: When one litre of gasoline (primarily octane) is combusted approximately 42 MJ of thermal energy are released. An internal combustion engine is mechanism that converts thermal energy into mechanical energy. Due to the Laws of Thermodynamics (specifically the Law that prevents entropy from decreasing) at most 30% of the thermal energy can be converted into mechanical energy. This mechanical energy is the work that increases the kinetic energy of the car. If 50 mL of gasoline is combusted, what final speed (in km/h) can a 1723 kg minivan achieve if it began at rest?

PROBLEM 4.2.03: A car ($m_c = 1320$ kg) traveling at 60 km/h comes to a stop. Its kinetic energy is converted by friction into thermal energy in the disc brakes. These are two circular discs steel ($\mathscr{C} = 466 \frac{\text{J}}{\text{kg}\cdot\text{C}^\circ}$), each of mass $m_d = 9.5$ kg. By what amount does their temperature increase?

PROBLEM 4.2.04: Water falls 15.0 m at a rate of 35.0 m^3/s. Only 60% of that water's kinetic energy can be captured and converted to electrical energy by our generators:
(a) What electrical power can be produced?
(b) If we sell the energy produced at 0.06 $/kWh, how much will we profit per day?

PROBLEM 4.2.05: How much does it cost to heat a 2.50 litre container of coffee from 4 °C to 99 °C using a 1200 W microwave? (Assume the coffee is like water with $\mathscr{C} = 4.184$ J/gram·°C, and that Hydro Quebec charges $0.0591/kWh.)

PROBLEM 4.2.06: A person is using a 2200 W heater to keep their apartment warm. After twenty minutes off, it turns on for five minutes, and that repeats. If they keep it running like this how much will it cost them for a 30-day month? (Hydro Quebec charges $0.0591/kWh.)

PROBLEM 4.2.07: One litre of gasoline, when combusted, can release 42 MJ (megajoules) of thermal energy. If my car used 20 litres of gas to drive to Ottawa, then
(a) How much thermal energy did my car produce over the whole trip?
(b) If the trip took 2.0 hours, what was the rate (in kilowatts) at which thermal energy was produced by the engine?
(c) If only 30% of that thermal energy was converted into mechanical work that actually moved my car, what was this mechanical power output, measured in horsepower?

PROBLEM 4.2.08: As a car drives down the road, its tires are turning. The total kinetic energy of the car is thus the sum of the linear kinetic energy of the body of the car plus the linear and angular kinetic energies of the tires. A wheel of radius R that is rolling at a speed v is rotating with an angular speed $\omega = v/R$.

The body of a car has mass 1091 kg. Each of its tires ($R = 29.4$ cm) has mass 13.6 kg and moment of inertia $\mathscr{I} = 0.667\,\text{kg}\cdot\text{m}^2$. If this car is traveling at 55 km/h, what is its total kinetic energy?

PROBLEM 4.2.09: A steam engine expels 81 J of energy to the environment for every 100 J of internal energy it uses. **(a)** How much work does it generate? **(b)** What is the engine's efficiency?

PROBLEM 4.2.10: A door is being closed. It measures 2.032 m tall by 76.2 cm wide, and has a mass of 11.340 kg. The door swings 90° in 1.66 s.
(a) What is the door's angular kinetic energy?
(b) A short push was given to make the door close, applied only while it swung through the first 15° of its motion. What force was applied (perpendicular) to the edge of the door?

PROBLEM 4.2.11: CHALLENGE: A fan of diameter 45 cm is moving air at 3.8 m/s. What is the power exerted by the fan to move the air at this rate? (The density of air is $\rho_{\text{air}} = 1.225\,\text{kg/m}^3$.)

4.3 Energy in Biological Contexts

In the context of nutrition the unit of energy is the Calorie:

$$1\,\text{Cal} = 4\,184\,\text{J} \tag{4.26}$$

This is the energy that will increase the temperature of one litre of water by $1\,\text{C}^\circ$ (approximately).

For each litre of oxygen that a person respires metabolic processes release approximately 20 kJ of energy that can be used by the body to perform mechanical work, or to maintain temperature.

A unit of power that is sometimes used is the "horsepower":

$$1\,\text{hp} = 746\,\text{W} \tag{4.27}$$

This power can be achieved (for very brief intervals) by elite athletes!

4.3.1 Units

EXR 4.3.01 What is 100 Cal expressed in kilowatt-hours?

EXR 4.3.02 What is half a horse-power expressed in Calories per minute?

EXR 4.3.03 What is 12.2 mL of Oxygen per second expressed in litres of Oxygen per minute?

EXR 4.3.04 What is the "$1\,\text{L}_{\text{O}_2} \to 20\,\text{kJ}$" relation expressed in Calories?

4.3.2 Metabolic Energy & Power

EXR 4.3.05 What is the rate of metabolic power output of a sitting person who consumes 0.30 L of oxygen per minute?

EXR 4.3.06 What is the rate of oxygen consumption during sleep assuming a metabolic rate of 75 W?

EXR 4.3.07 If a person eats 1 670 Cal a day, what power (measured in watts) are they generating, on average?

EXR 4.3.08 If a person's metabolic rate is 75 W while they are sleeping (for eight hours), but their average metabolic rate over a 24 h period is 81 W, then what is their average metabolic rate while they are awake?

4.3.3 Biomechanical Energy, Work & Power

EXERCISE 4.3.09 While a person is walking one of their legs swings forward through an angle of 30° in 0.75 s. If their leg has a moment of inertia $\mathscr{I} = 1.83\,\text{kg}\cdot\text{m}^2$, what is the angular kinetic energy of their leg?

EXERCISE 4.3.10 A person is waving their arm through an angle of 42°, one side to the other, four times in 1.08 s. If their arm has a moment of inertia $\mathscr{I} = 0.583\,\text{kg}\cdot\text{m}^2$, what is the angular kinetic energy of their arm?

EXERCISE 4.3.11 A person opening a door does 1.305 J work to it (changing its motion from being at rest to swinging open). Their upper arm rotates about their shoulder 107° while pulling on the door. If the muscles in the shoulder complex can be modeled as a single force acting along a line 20.8 mm from the shoulder joint, what is the tension in these muscles?

EXERCISE 4.3.12 A person performs 0.103 kWh of work during their day. If the efficiency of the food-to-work process in their body is only 18%, how many Calories did their body consume to perform this work?

4.3.4 Problems

EXERCISE 4.3.13 A person who is standing upright lifts one of their arms (3.5 kg) from straight at their side to straight over their head. During this motion the center of mass of this segment is raised 55 cm. If they repeat this motion 40 times, and the efficiency of their internal energy conversion is 18%, how many calories will this require?

PROBLEM 4.3.01: One serving of Pad Thai contains about 573 Calories of food energy (1 Calorie = 4.184 kJ). If you have a body mass of 65 kg, when you walk at 9 km/h you "burn" 9.55 Calories per minute.
(a) What is the rate of energy expenditure, measured in watts?
(b) At this rate how long (in hours) would it take to "burn off" your Pad Thai?

PROBLEM 4.3.02: Assuming that muscles convert food energy into mechanical energy with an efficiency of 22%, how much food energy is converted by an 80-kg man climbing a vertical distance of 15m? Express your answer in kilojoules, and in food Calories.

PROBLEM 4.3.03: A grocery store worker is placing pop bottles on a shelf. Every 24 s they place ten 2 L bottles on the shelf, raising them 72 cm from the delivery pallet to the shelf. If they continue this job, at this rate, for 13 minutes:
(a) What is the total work done?
(b) How many Calories did they "burn"? (Assume $\mathscr{E} = 0.20$.)

PROBLEM 4.3.04: An 80-kg man runs up stairs, ascending 6.0 m in 8.0 s. What is his power output in kilowatts, and in horsepower?

PROBLEM 4.3.05: The chemical process that drives muscles can produce about 20 kJ of mechanical energy for each litre of oxygen a human respires. If a sprinter consumes 4.1 L/min of oxygen, what is their maximum possible power output, in watts, and in Calories per minute?

PROBLEM 4.3.06: A 64kg woman is slowly descending the stairs, traveling 16.5m downwards in 37s. **(a)** At what rate must her body dissipate gravitational energy so that so descends at a constant rate? **(b)** If all of that energy was converted into thermal energy in her body, by what amount would her temperature increase?

PROBLEM 4.3.07: In the video "Olympic Cyclist vs Toaster", Robert Förstemann (Germany) sustains a power output of 700W for one minute, approximately. (Notice: That is almost one horse-power!) **(a)** What was his total energy output, measured in kilowatt-hours? **(b)** If that output was only 22% of the chemical energy consumed by his muscles, how many Calories did that effort require? **(c)** Consequently, how much oxygen did he need to respire?

PROBLEM 4.3.08: While a person is walking one of their leg swings forward through an angle of 30° in 0.75 s. Their leg has a moment of inertia $\mathscr{I} = 1.83\,\text{kg}\cdot\text{m}^2$. At the beginning of the swing their leg has $\omega = 0\,\text{rad/s}$, and the maximum occurs half-way through the swing, where ω equals *twice* the average angular speed across the whole motion. Assuming the moment arm between the muscles and the hip joint is approximately 5 cm, what it the tension in the muscles during this motion?

PROBLEM 4.3.09: A person nodding their head ("yes") moves their head 15° up-and-down five times in 1.83 s. For this rotation their head has a moment of inertia $\mathscr{I} = 0.0847\,\text{kg}\cdot\text{m}^2$. The muscles at the back of their neck act along a line 43.8 mm from the joint at the base of their skull. What is the tension in these muscles? (Note: At the beginning of the "nod" head leg has $\omega = 0\,\text{rad/s}$, and the maximum occurs half-way through the motion, where ω equals *twice* the average angular speed across the whole motion.)

Chapter 5

Waves

5.1 Periodic Waves

5.1.1 Period & Frequency

In these exercises we will be determining the period T (the amount of time between repetitions) and the frequency f (the rate of repetition) of various phenomena. The frequency f of a repeating event is defined to be the ratio of the number of repetitions to the time taken for those repetitions to occur, and is measured in hertz:

$$1\,\text{hertz} = 1\,\frac{\text{repetition}}{\text{second}} \tag{5.1}$$

$$1\,\text{Hz} = 1\,\frac{\text{rep}}{\text{s}} \tag{5.2}$$

(Note that "repetition" is not an SI unit, but is a place-holder or reminder. When calculating it does not contribute anything except to help keep track of consistency. Strictly speaking $1\text{ Hz} = 1\text{ s}^{-1}$.) Because of their definitions they relate: $f = 1/T$. So we can also write the units of period as "seconds per repetition".

Exr 5.1.01 If a person's heart rate is 70 beats per minute, what are the period and frequency of that oscillation?

Exr 5.1.02 The human heart rate can fall in the range from 60 beats per minute (very relaxed) to 120 beats per minute (intense exercise). To what range of frequencies does this correspond?

Exr 5.1.03 An adult human at rest has a respiration rate between 12 to 16 times a minute. To what range of frequencies does this correspond?

Exr 5.1.04 During exercise a person is respiring at a rate of 40 breaths per minute. What is period and frequency of this rate?

Exr 5.1.05 The average computer user can type about 190 characters per minute, while professional typists can type as fast as 360 characters per minute. What are the frequency of keys per second for these two keyboardists?

Exr 5.1.06 Players of video games sometimes need to perform "button mashing" when they repeatedly press a control a rapidly as possible. A player pressed a button 191 times in 12 seconds. What are the period and frequency of this rate of button mashing?

Exr 5.1.07 The average male voice has an oscillation around 130 Hz. A bass singer usually has a low note around 82.4 Hz. One of the lowest bass singers, a fellow by the name Tim Storms, can sing notes lower than B0 (30.87 Hz)! What are the periods of oscillation of the vocal chords of men making these three notes?

Exr 5.1.08 The average female voice has an oscillation around 210 Hz. A soprano singer usually has a high note around 650 Hz. Above the soprano range is the so-called *whistle register*. Mariah Carey was once recorded hitting G7 (3.136 kHz)! What are the periods of oscillation of the vocal chords of women making these three notes?

Exr 5.1.09 The heart of a hummingbird can beat as fast as 1260 times a minute. What is this rate measured in hertz?

Exr 5.1.10 The wings of a hummingbird can flap as fast as 10 000 times in three minutes! What is this rate measured in hertz?

Exr 5.1.11 The musical-sounding notes produced by birds, *birdsong*, falls in the range 1 kHz to 8 kHz. (Compare this with the whistle register of human singing.) What is the range of the periods of oscillation of the vocal chords of birds making notes in this range?

Exr 5.1.12 Lower-frequency sounds travel further than high-frequency sounds. (This is due to the phenomena of *attenuation* that we will study later.) Elephants communicate using some frequencies below what humans can hear. The tones they produce fall in the range of 5 Hz to 30 Hz. What is the range of the periods of oscillation corresponding to notes in this range?

5.1.2 The Fundamental Relationship for Periodic Waves

These exercises will use the fundamental relationship for periodic waves

$$\lambda = v/f \tag{5.3}$$

to relate Frequency (f), Wavelength (λ) and Wavespeed (v). The SI units to use for these quantities are hertz (Hz) for frequency, metres (m) for wavelength, and metres per second (m/s) for wavespeed.

Exr 5.1.13 What is the speed of a periodic wave whose frequency and wavelength are 500 Hz and 0.5 m respectively?

Exr 5.1.14 What is the wavespeed on a steel cable if a periodic wave of frequency 75 Hz has wavelength 1.333 m?

Exr 5.1.15 What is the speed of an ultrasonic wave in flesh whose frequency and wavelength are 2.20 MHz and 705 µm respectively?

Exr 5.1.16 What is the speed of a ultrasonic wave in air whose frequency and wavelength are 45.0 kHz and 7.63 mm respectively?

Exr 5.1.17 What is the wavelength of a periodic wave on a stretched spring whose wavespeed and period of oscillation are are 75 m/s and 0.005 s respectively?

Exr 5.1.18 Electrical power is delivered by oscillating voltages of frequency 60.0 Hz. These electrical oscillations can sometimes cause mechanical oscillations of equal frequency, which then become sound in the surrounding air. Since the wavespeed in air is 343 m/s what is the wavelength of these sounds?

Exr 5.1.19 What is the wavelength of an underwater sound wave whose wavespeed and frequency of oscillation are are 1480 m/s and 5.90 kHz respectively?

Exr 5.1.20 Some ultrasonic waves traveling through fat tissue have wavespeed and frequency of oscillation 1450 m/s and 1.55 MHz respectively. What is the wavelength of these waves?

Exr 5.1.21 What is the frequency of a periodic wave whose wavespeed and wavelength are 120 m/s and 30 cm respectively?

Exr 5.1.22 What is the frequency of a wave on a stretched spring whose wavespeed and wavelength are 3.91 m/s and 32.5 cm respectively?

Exr 5.1.23 What is the frequency of a sound wave whose speed and wavelength are 343 m/s and 20.3 cm respectively?

Exr 5.1.24 What is the frequency of an ultrasonic wave traveling through bone whose speed and wavelength are 3290 m/s and 1.94 mm respectively?

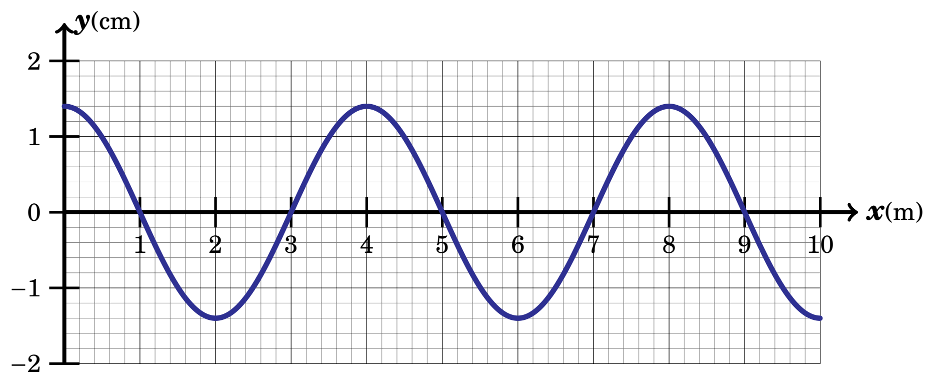
5.1.3 Graphs of Waves

Determining the Amplitude, Wavelength, and other parameters, of Pulses and Periodic Waves from Graphs.

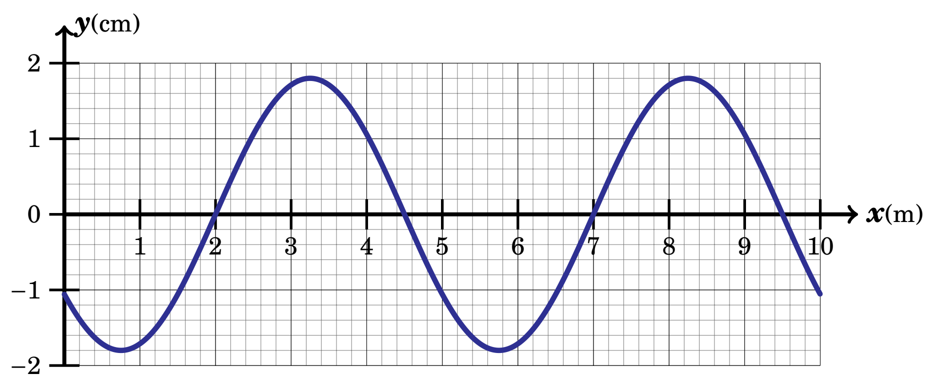
Terminology: A *pulse* is a wave that does not repeat. A pulse travels at the wavespeed of the medium, but does not have a wavelength or frequency because it does not repeat across space or across time.

Displacement versus Position

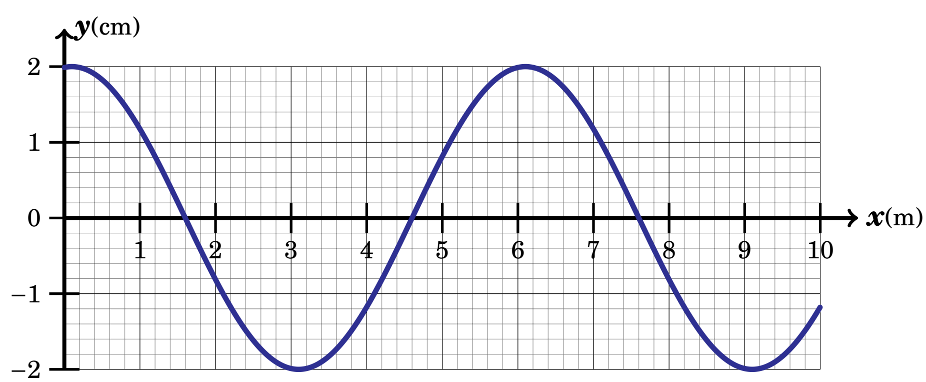
Exr 5.1.25 What are the amplitude and wavelength of the periodic wave graphed below?



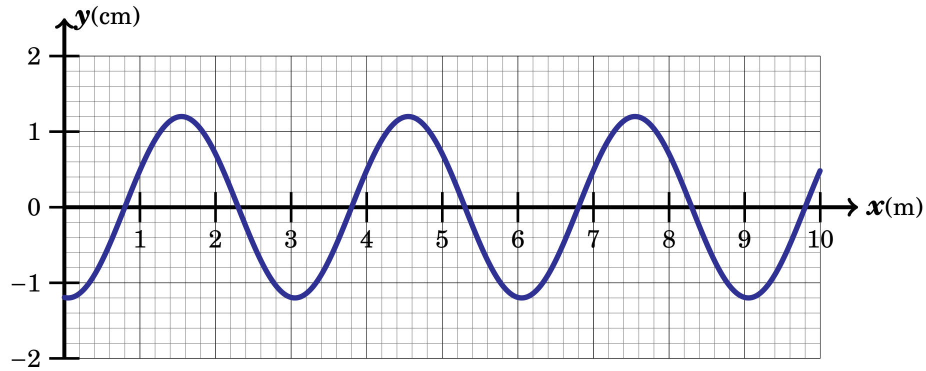
Exr 5.1.26 What are the amplitude and wavelength of the periodic wave graphed below?



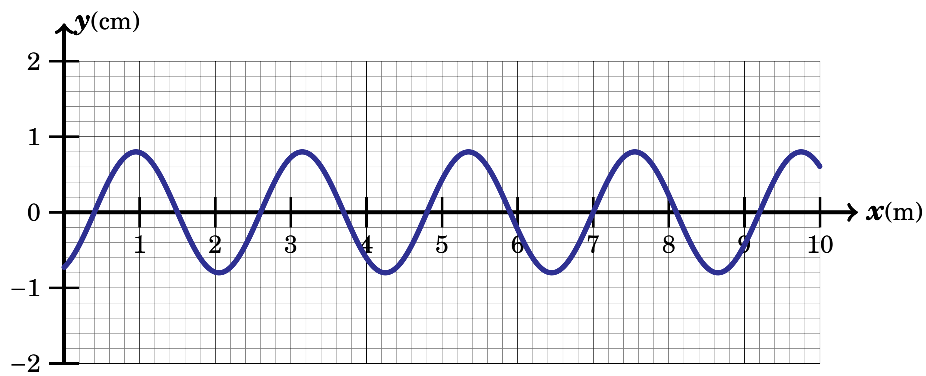
Exr 5.1.27 What are the amplitude and wavelength of the periodic wave graphed below?



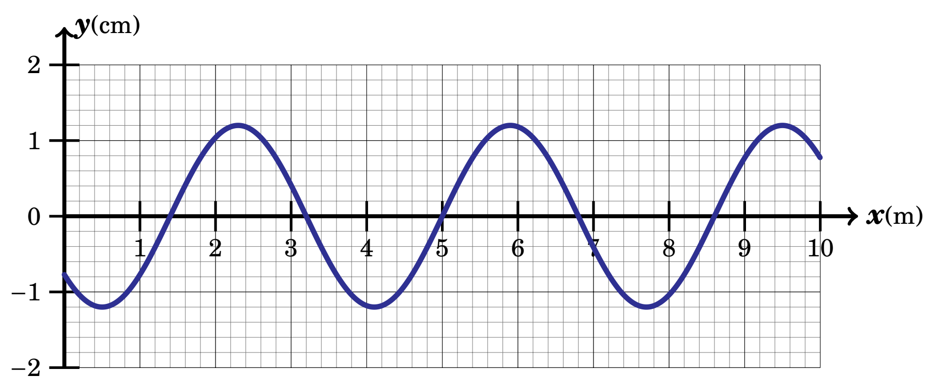
EXR 5.1.28 What are the amplitude and wavelength of the periodic wave graphed below?



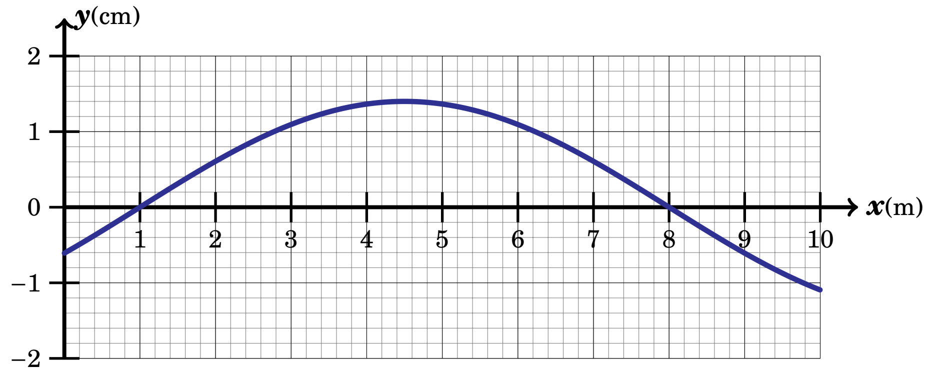
EXR 5.1.29 What are the amplitude and wavelength of the periodic wave graphed below?



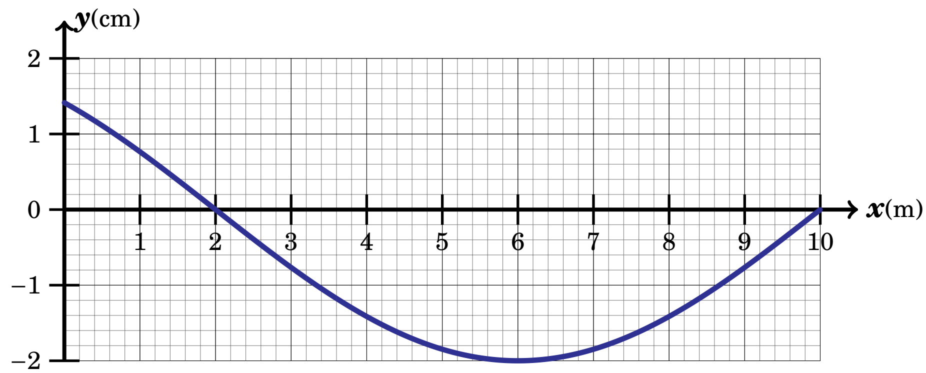
EXR 5.1.30 What are the amplitude and wavelength of the periodic wave graphed below?



Exr 5.1.31 What are the amplitude and wavelength of the periodic wave graphed below?

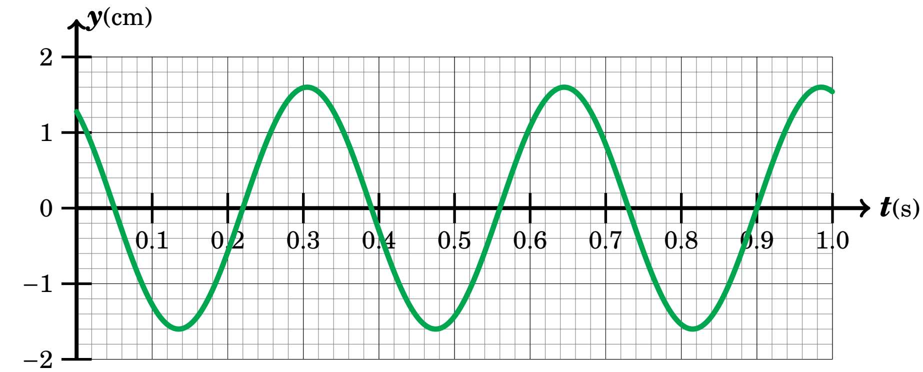


Exr 5.1.32 What are the amplitude and wavelength of the periodic wave graphed below?

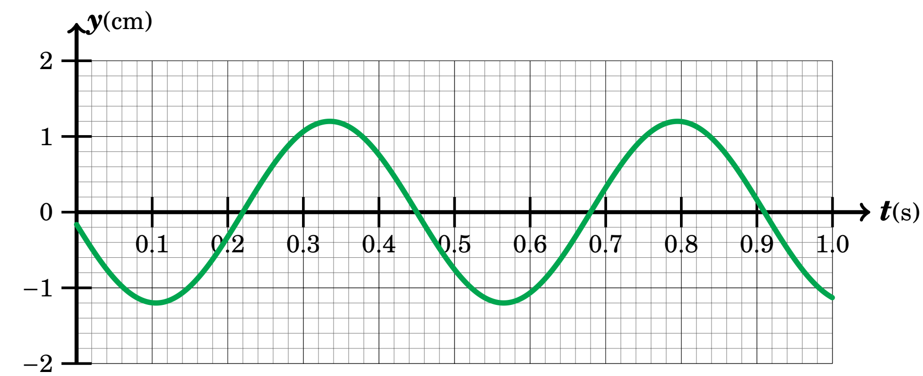


Displacement versus Time

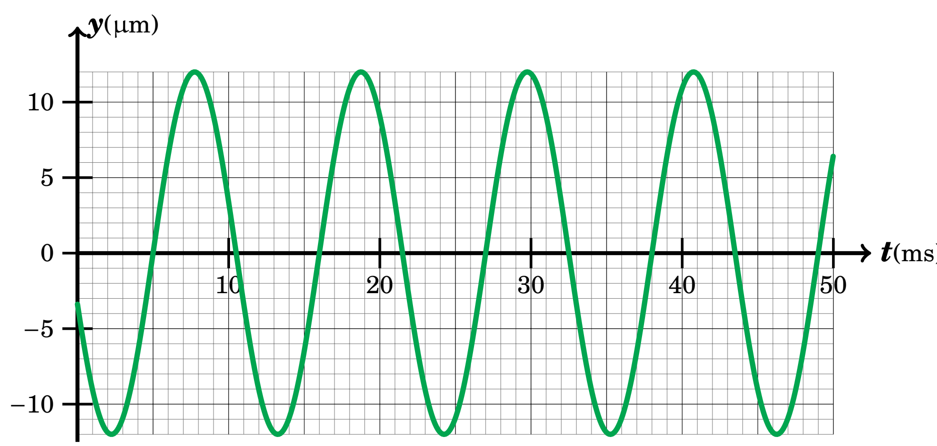
Exr 5.1.33 A periodic wave is traveling across a stretched string. Graphed below is the displacement as a function of *time* of a single piece of that string. What are the amplitude, period, and frequency of this wave?



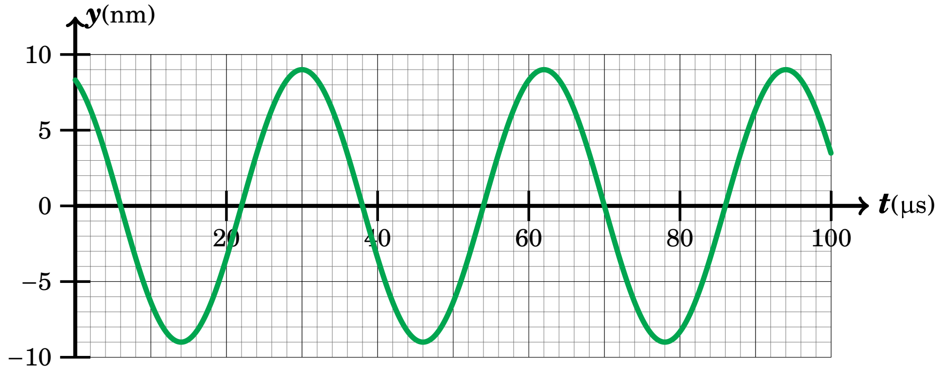
Exr 5.1.34 A periodic wave is traveling across a stretched string. Graphed below is the displacement as a function of time of a single piece of that string. What are the amplitude, period, and frequency of this wave?



Exr 5.1.35 A periodic wave is traveling across a metal rod. Graphed below is the displacement as a function of time of a single piece of that object. What are the amplitude, period, and frequency of this wave?

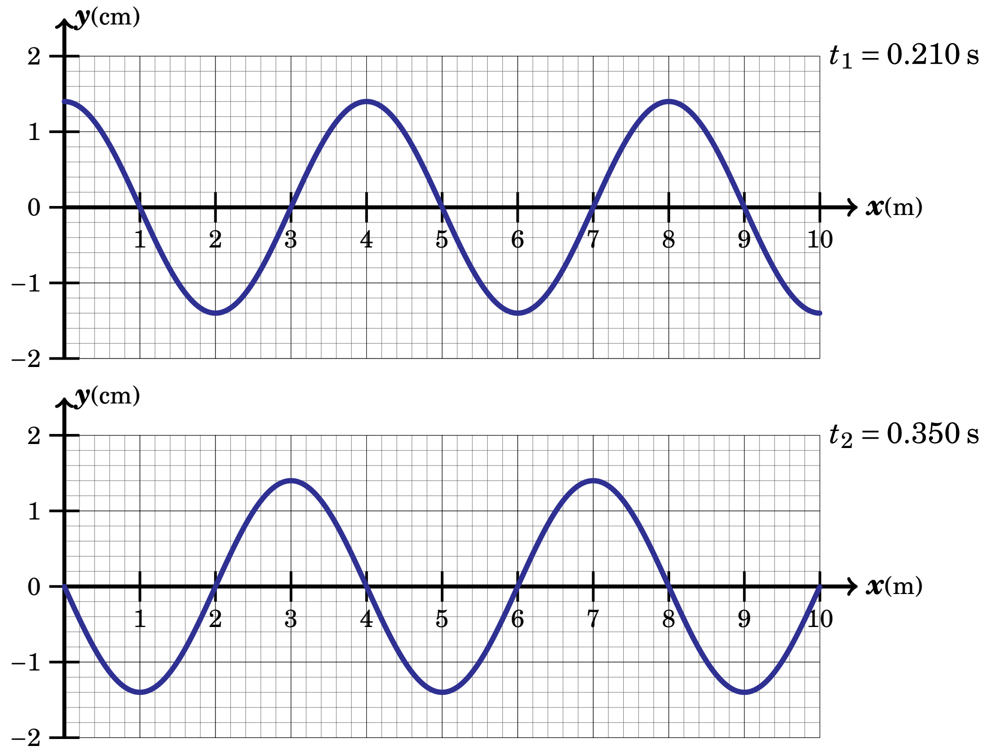


EXR 5.1.36 A periodic sound wave is traveling through water. Graphed below is the displacement as a function of time of a single portion of the medium. What are the amplitude, period, and frequency of this wave?

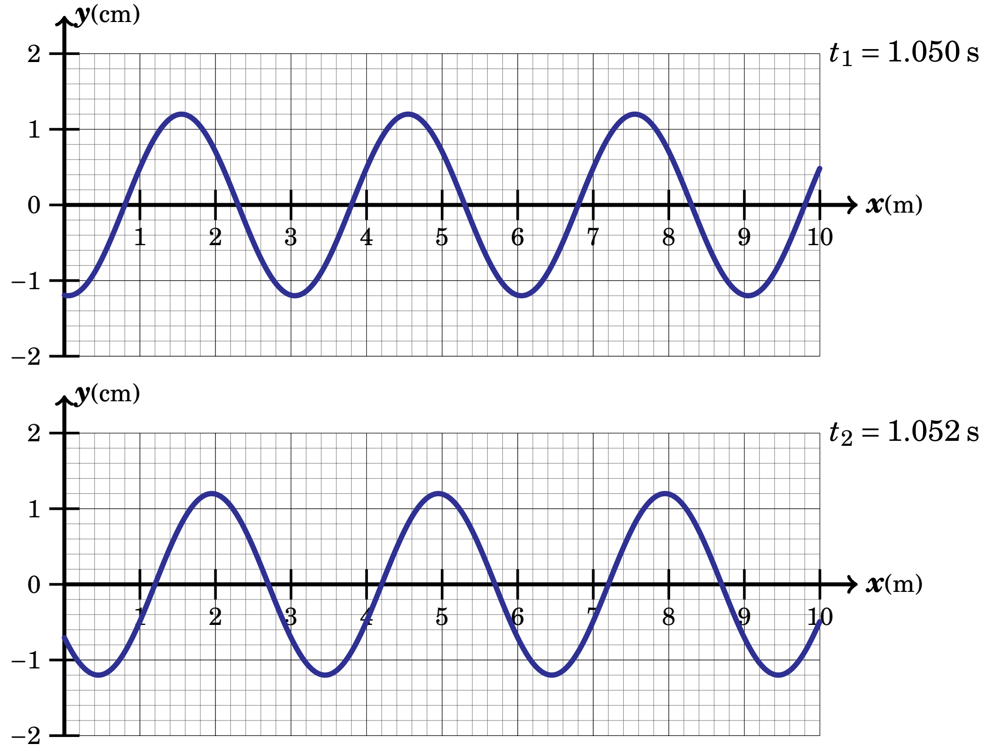


Pairs of Graphs

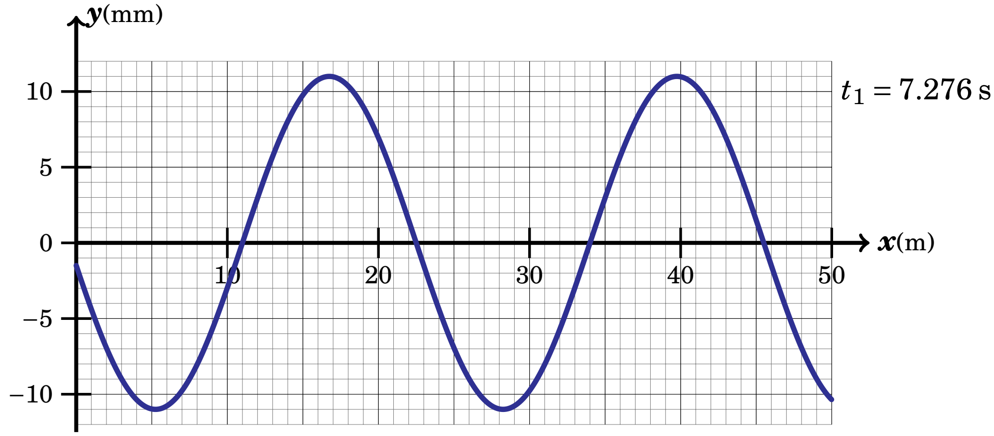
PROBLEM 5.1.01: Below are two photographs of a traveling periodic wave, taken at two different times. (The time between these photographs is less than half the period of oscillation of the source.) **(a)** What direction is the wave traveling, and at what speed? **(b)** What are the wavelength and frequency of this wave?

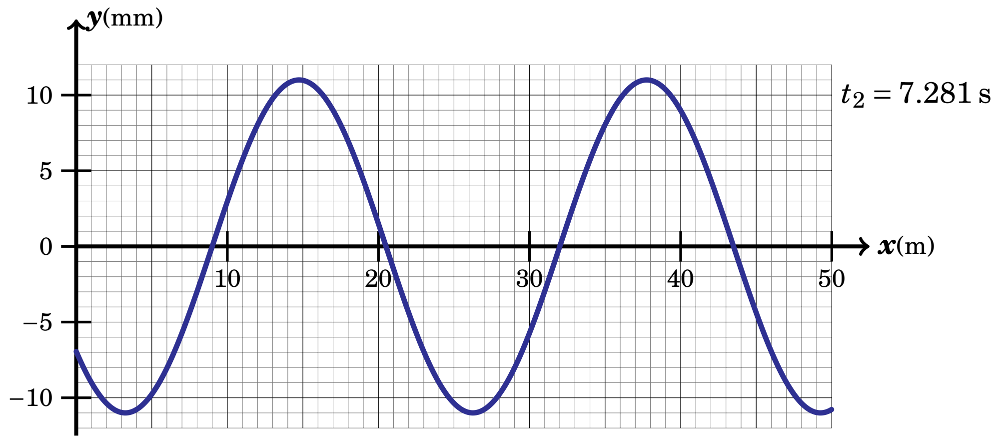


PROBLEM 5.1.02: Below are two frames from a video of a traveling periodic wave. (The time between these photographs is less than half the period of oscillation of the source.) **(a)** What direction is the wave traveling, and at what speed? **(b)** What are the wavelength and frequency of this wave?



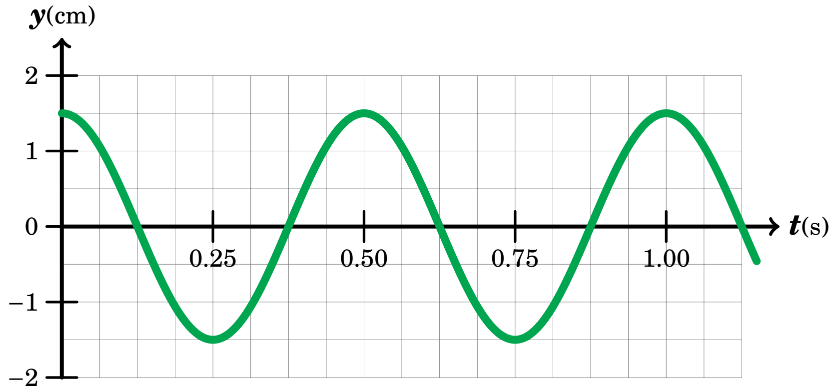
PROBLEM 5.1.03: Below are two frames from a video of a periodic wave traveling along a thin metal rod. (The time between these photographs is less than half the period of oscillation of the source.) **(a)** What direction is the wave traveling, and at what speed? **(b)** What are the wavelength and frequency of this wave?

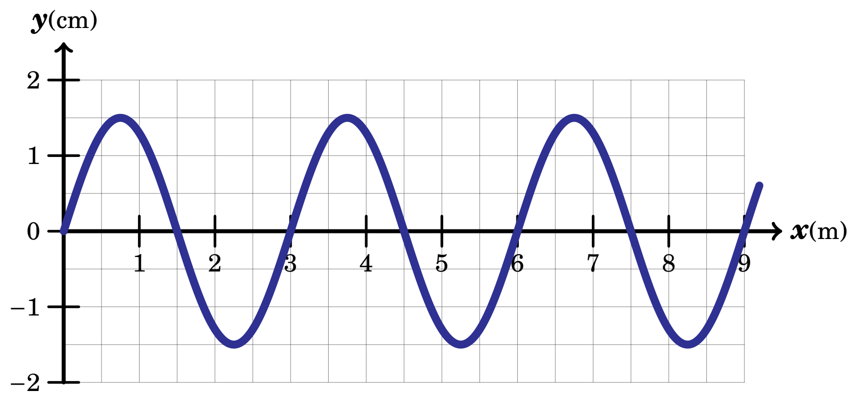




PROBLEM 5.1.04: The graphs below show measurements made of a traveling wave on a string. The first graph is the displacement (y) as a function of time (t) for a small piece of the string. The second graph is the displacement (y) as a function of position (x) along the length, taken at $t = 0\,\text{s}$.

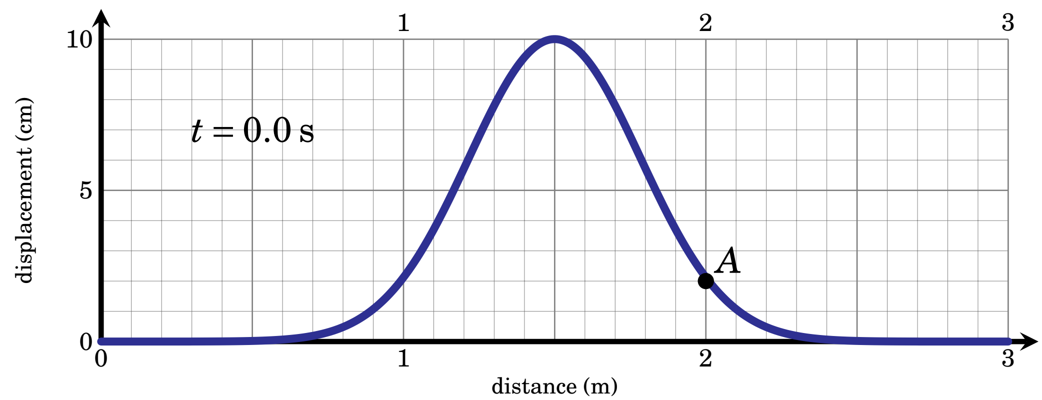
Find **(a)** the amplitude, **(b)** the wavelength, **(c)** the period, **(d)** the frequency, and **(e)** the wavespeed of the wave.

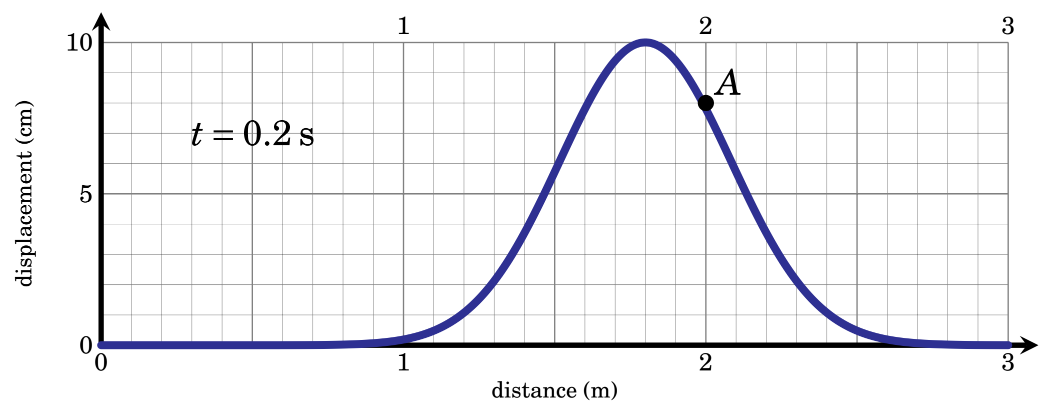




5.1.4 Problems

PROBLEM 5.1.05: The figures below show a pulse on a string at two times: $t = 0$ s and $t = 0.2$ s. Since this is a pulse, not a continuous periodic wave, so there is neither a "wavelength" nor a "period" of oscillation. (Note carefully that the horizontal and vertical scales are different.) **(a)** What is the speed of the pulse? **(b)** What is the vertical speed of the piece of string labeled "A" during this interval? **(c)** What is the position of the peak of the pulse at $t = 3.0$ s? **(d)** At what time will the pulse arrive at the $x = 4.0$ m position?
Answers: **(a)** $v = 1.5$ m/s; **(b)** $v = 0.30$ m/s; **(c)** $x = 6.0$ m; and **(d)** $t = 1.67$ s.

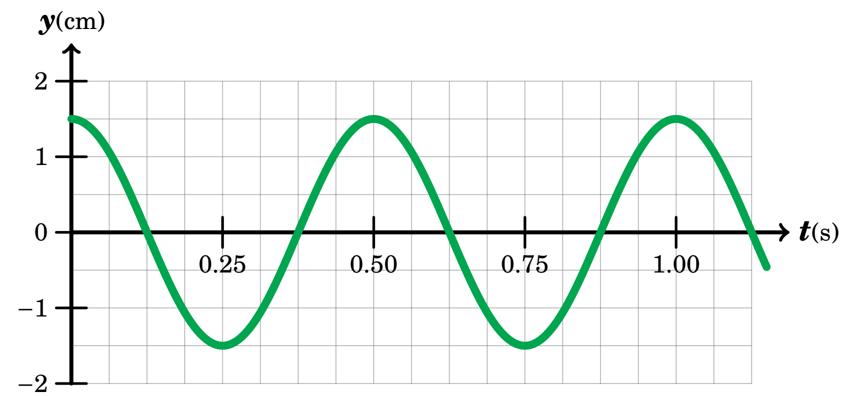


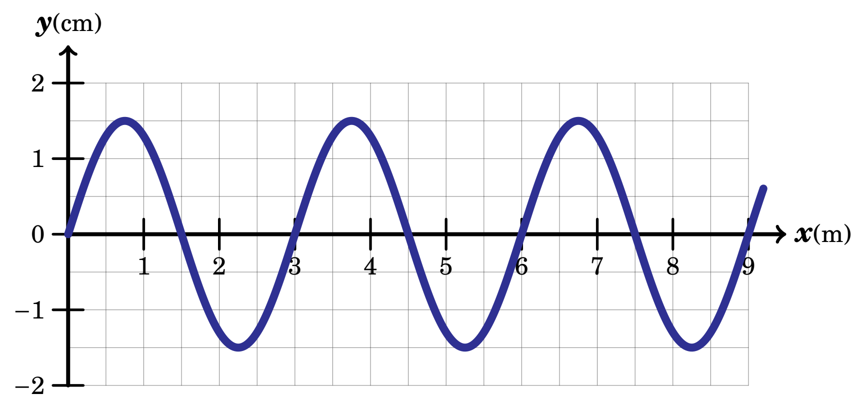


Problem 5.1.06: The graphs below show measurements made of a traveling wave on a string. The first graph is the displacement (y) as a function of time (t) for a small piece of the string. The second graph is the displacement (y) as a function of position (x) along the length, taken at $t = 0$ s.

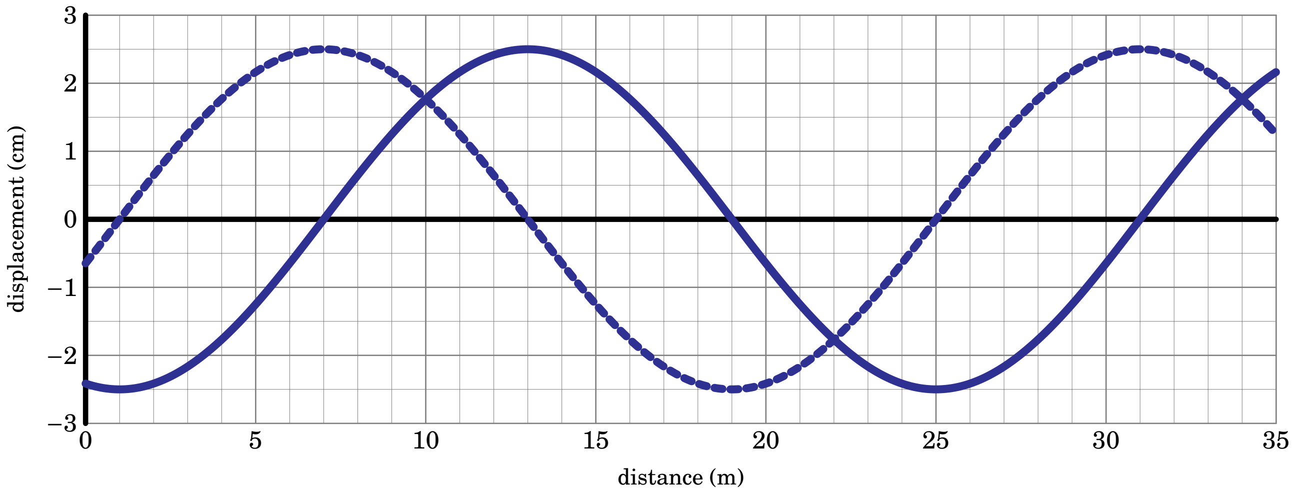
Find **(a)** the amplitude, **(b)** the wavelength, **(c)** the period, **(d)** the frequency, and **(e)** the wavespeed of the wave.

Answers: **(a)** $A = 1.5$ cm; **(b)** $\lambda = 3.0$ m; **(c)** $T = 0.500$ s; **(d)** $f = 2.00$ Hz; and **(e)** $v = 6.0$ m/s.





PROBLEM 5.1.07: The graph below shows a sine wave traveling to the right on a string. The dashed line is the shape of the string at time $t = 0$ s, and the solid curve is the shape of the string at time $t = 0.12$ s. (Note carefully that the horizontal and vertical scales are different.) Find **(a)** the amplitude, **(b)** the wavelength, **(c)** the speed, **(d)** the frequency, and **(e)** the period of the wave.
Answers: **(a)** $A = 2.5$ cm; **(b)** $\lambda = 24$ m; **(c)** $v = 50$ m/s; **(d)** $f = 2.08$ Hz; and **(e)** $T = 0.48$ s.



5.2 Sound

Traveling waves in air, water and people.

5.2.1 Intensity, Power, and Area

Intensity measures the way in which the power transferred by a wave is distributed across an area:

$$I = P/A \tag{5.4}$$

Intensity is typically measured in watts per square metre W/m^2, but in some circumstances (when the areas involved are human-sized) it is measured in watts per square centimetre W/cm^2. It is important to remember that while lengths convert as 1 m=100 cm, areas convert as $1\,\text{m}^2 = (100\,\text{cm})^2 = 10\,000\,\text{cm}^2$, so that $1\,\text{cm}^2 = 10^{-4}\,\text{m}^2$. In all cases pay careful attention to the units of the lengths and the units of the area(s) involved.

When a wave spreads out in three dimensions, the wavefronts are spherical. The surface area of a sphere is given by

$$A_{\text{sphere}} = 4\pi r^2 \tag{5.5}$$

(Yes, this differs from the formula for the area of a flat circle by a factor of four, so be careful not to mix them up.) Because of this, the intensity a distance r from a source of power P (producing either sound or light) is

$$I = \frac{P}{4\pi r^2} \tag{5.6}$$

This is known as the *inverse-square law* for intensity.

Exr 5.2.01 Find the intensity (measured in W/m^2) of a wave that delivers 16.0 W to a rectangular area that measures 23 cm by 71 cm.

Exr 5.2.02 Find the intensity (measured in W/m^2) of a wave that delivers 72.5 W to the surface of a door that measures 1.05 m by 1.82 m.

Exr 5.2.03 Find the intensity (measured in W/m^2) of a wave that delivers 60.0 W to a rectangular area that measures 2.50 m by 3.75 m.

Exr 5.2.04 Find the intensity (measured in W/m^2) on a soccer field (measuring 73 m by 110 m) illuminated by 7.50 kW of lighting.

Exr 5.2.05 Find the intensity (measured in W/cm^2) of a wave that delivers 90.4 W to a rectangular area that measures 7.5 cm by 11.0 cm.

Exr 5.2.06 Find the intensity (measured in W/cm^2) of a wave that delivers 72.5 W to the surface of a door that measures 1.05 m by 1.82 m.

Exr 5.2.07 Find the intensity (measured in W/cm^2) of light that delivers 15.0 W onto a piece of paper (measuring 210 mm by 297 mm).

Exr 5.2.08 Find the intensity (measured in W/cm^2) of a computer monitor that emits 0.500 W of light energy. (measuring 274 mm by 487 mm).

Exr 5.2.09 Find the intensity (measured in W/m^2) of a wave that delivers 13.7 W to a circular area 5.00 m in diameter.

Exr 5.2.10 Find the intensity (measured in W/m^2) of a wave that delivers 88 mW to a circular area 75.2 cm in diameter.

Exr 5.2.11 Find the intensity (measured in W/cm^2) of a wave that delivers 0.42 W to a circular area 6.2 cm in diameter.

Exr 5.2.12 Find the intensity (measured in W/cm^2) of a wave that delivers 72 W to a circular area 3.20 m in diameter.

Exr 5.2.13 Find the intensity of a wave measured 5.00 m from a 4.20 W source.

Exr 5.2.14 Find the intensity of a wave measured 5.42 m from a 0.37 W source.

Exr 5.2.15 Find the intensity (measured in units of W/cm^2) of a wave measured 6.2 cm from a 17.4 W source.

Exr 5.2.16 Find the intensity (measured in units of W/cm^2) of a wave measured 2.56 m from a 0.909 W source.

5.2.2 Intensity, Energy, and Time

Since power relates to energy and time, we can relate intensity to the amount of energy that was transferred:

$$P = I \times A \tag{5.7}$$

$$\Delta E = P \times \Delta t = I \times A \times \Delta t \tag{5.8}$$

These relations can also allow us to solve for the time required to deliver a required amount of energy:

$$\Delta t = \Delta E \big/ (I \times A) \tag{5.9}$$

If the source produces a wave whose amplitude is modulated, then the time required will be longer than that given by the equation above. The most common is when the source operates on a *duty cycle*, alternating between being on then off repeatedly. If $\delta \leq 1$ is the fraction of time that the source is "on", then $\Delta t_{\text{on}} = \delta \times \Delta t_{\text{total}}$, so that

$$\Delta t_{\text{on}} = \Delta E \big/ (I \times A) \tag{5.10}$$

$$\Delta t_{\text{total}} = \Delta t_{\text{on}} \big/ \delta \tag{5.11}$$

where I is the intensity of the wave when the source is "on".

Where necessary recall that $1\,\text{Cal} = 4\,184.\,\text{J}$ is the nutritional calorie, and that $1\,\text{kWh} = 3.6\,\text{MJ}$ is the usual unit of hydro-electrical energy (where each of these conversions are exact numbers).

Exr 5.2.17 Find the energy (measured in J) delivered by a wave of intensity $2.11\,\text{W/m}^2$ to a rectangular area that measures 1.24 m by 75 cm after 51 seconds.

Exr 5.2.18 Find the energy (measured in J) delivered by a wave of intensity $37.0\,\text{mW/m}^2$ to an area of $4.00\,\text{m}^2$ after one hour.

Exr 5.2.19 Find the energy (measured in kWh) delivered by a microwave oven (intensity $2.560\,\text{kW/m}^2$) to an area measuring 28 cm by 23 cm after 33 minutes.

Exr 5.2.20 Find the energy (measured in kWh) delivered by a wave of intensity $13.0\,\text{W/m}^2$ to an area of $15.0\,\text{m}^2$ after eight hours.

Exr 5.2.21 Find the energy (measured in J) delivered by a wave of intensity $0.64\,\text{W/cm}^2$ to a rectangular area that measures 23 cm by 71 cm after 5 minutes.

Exr 5.2.22 Find the energy (measured in J) delivered by a wave of intensity $0.053\,\text{W/cm}^2$ to a rectangular area that measures 17 cm by 20 cm after 3 minutes and 12 seconds.

Exr 5.2.23 Find the energy (measured in Cal) delivered by a wave of intensity $12.0\,\text{W/cm}^2$ to an area of $200\,\text{cm}^2$ after seven and a half minutes.

Exr 5.2.24 Find the energy (measured in Cal) delivered by a wave of intensity $9.50\,\text{W/cm}^2$ to a circular area of diameter 18.0 cm after fifteen minutes.

Exr 5.2.25 Find the time required (measured in minutes and seconds) to deliver 513 kJ over a rectangular area measuring 20.6 cm by 19.4 cm using a source with intensity $32\,\text{W/cm}^2$.

Exr 5.2.26 Find the time required (measured in minutes and seconds) to deliver 1.20 MJ over a square area measuring 0.50 m on each side using a source with intensity $6.4\,\text{W/cm}^2$.

Exr 5.2.27 Find the time required (measured in minutes and seconds) to deliver 0.765 Cal over a rectangular area measuring 21 cm by 23 cm using a source with intensity $5.23\,\text{mW/cm}^2$.

Exr 5.2.28 Find the time required (measured in minutes and seconds) to deliver 82.0 Cal over a circular area of diameter 15.0 cm using a source with intensity $7.00\,\text{W/cm}^2$.

Exr 5.2.29 Find the time required (measured in minutes and seconds) to deliver 513 kJ over a rectangular area measuring 20.6 cm by 19.4 cm using a source with intensity $32\,\text{W/cm}^2$ operating on a 20% duty cycle.

Exr 5.2.30 Find the time required (measured in minutes and seconds) to deliver 62.0 kJ over a circular area 8.7 cm in diameter using a source with intensity $17.3\,\text{W/cm}^2$ operating on a 10% duty cycle.

5.2.3 Sound Level

The quantitative measure β that models subjective loudness is ***sound level***, defined by:

$$\beta = (10\,\text{dB}) \log\left(I/I_0\right) \tag{5.12}$$

The logarithm returns a number, and the factor 10 dB expresses the level as a multiple of the unit decibel (dB) that measures level. The argument of the logarithm is the ratio of the intensity I of the sound wave to I_0 the *reference intensity* which is defined to be the exact quantity

$$I_0 = 1 \times 10^{-12}\,\text{W/m}^2 \tag{5.13}$$

which corresponds (roughly) to the quietest sound that an average person can hear.

In cases where the level is known and the intensity is asked for, the relation is

$$I = I_0 \times 10^{\beta/10\,\text{dB}} \tag{5.14}$$

Take care with the exponents in these calculations! Ideally you will figure out how to get your calculator to express all quantities in *scientific notation*.

Exr 5.2.31 Find the sound level (in decibels) of a sound that has intensity $1 \times 10^{-6}\,\text{W/m}^2$.

Exr 5.2.32 Find the sound level (in decibels) of a sound that has intensity $7.5 \times 10^{-8}\,\text{W/m}^2$.

Exr 5.2.33 Find the intensity (measured in $\mu\text{W/m}^2$) of a sound of level 65 dB.

Exr 5.2.34 Find the intensity (measured in $\mu\text{W/m}^2$) of a sound of level 53 dB.

Problem 5.2.01: Find the time required (measured in minutes and seconds) to deliver 1.37 μJ over an area of $21.0\,\text{cm}^2$ using a source with a sound level of 70 dB.

Problem 5.2.02: Find the energy delivered (measured in microjoules) over a circular area of diameter 14.7 cm using a source with a sound level of 80 dB after two and a half minutes.

Problem 5.2.03: Find the energy delivered (measured in microjoules) over a circular area of $0.60\,\text{cm}^2$ using a source with a sound level of 80 dB after three minutes.

Problem 5.2.04: Ultrasound waves can have very large intensities without posing a danger to human hearing since no portion of the inner ear responds to such high frequencies. If we have such a wave, of intensity $10^{+5}\,\text{W/m}^2$, what would be:

(a) The sound level of the wave?

(b) The total energy delivered to $1\,\text{cm}^2$ after one minute?

5.2.4 Sound Attenuation

The energy content of sound waves decreases with distance traveled due to internal friction in the medium. This process is called ***attenuation***. Attenuation relates the distance traveled by the wave (Δx) to the change in sound level ($\Delta\beta$):

$$\alpha = -\Delta\beta/\Delta x \tag{5.15}$$

The constant α is the attenuation coefficient. It is a property of the medium that depends upon the frequency of the wave. In the context of sound waves traveling through air α is usually measured in units of dB/km. In the context of ultrasound traveling through water or the human body α is usually measured in units of dB/cm.

Calculating energy deposition from attenuation: A drop in intensity to a fraction $I_2/I_1 = s < 1$ leads to a change in level by $\Delta\beta = (10\text{dB})\log s$. Since $s < 1$ and $\log s < 0$, it follows that $\Delta\beta < 0$ dB.

Exr 5.2.35 If the sound level of a wave has decreased by 7.5 dB after traveling 31 cm, what is the attenuation coefficient measured in dB/cm?

Exr 5.2.36 The sound level of a wave traveling through air decreases by 80 dB due to attenuation after traveling 333 m. What is the attenuation coefficient measured in dB/km?

Exr 5.2.37 Ultrasound waves (f = 4 MHz) passing through muscle decrease by 13.7 dB after traveling 2.71 cm. What is the attenuation coefficient measured in dB/cm?

Exr 5.2.38 If a wave is propagating through a material with an attenuation coefficient of 5.4 dB/cm, by what amount will its sound level have decreased after propagating 3.3 cm?

Exr 5.2.39 Ultrasonic waves of frequency 1 MHz propagates through air with an attenuation coefficient of 12.0 dB/cm. By what amount will the wave's sound level have decreased after propagating 2.20 mm?

Exr 5.2.40 Ultrasonic waves of frequency 10 MHz propagates through water with an attenuation coefficient of 0.232 dB/cm. By what amount will the wave's sound level have decreased after propagating 43.2 cm?

Exr 5.2.41 If a wave is propagating through a material with an attenuation coefficient of 8.0 dB/cm, what distance would it have to propagate before its sound level has decreased by 37 dB?

Exr 5.2.42 Ultrasound at 3 MHz propagates through muscle tissue with an attenuation coefficient of 4.15 dB/cm. What depth will it penetrate before its sound level has decreased by 22.0 dB?

Exr 5.2.43 High-frequency audible sound waves of 10 kHz propagates through air with an attenuation coefficient of 190 dB/km when the humidity is 40%. What distance would sound of that frequency have to travel before its sound level had decreased by 10.0 dB due to attenuation alone?

5.2.5 Problems

PROBLEM 5.2.05: An ultrasound device is set to produce waves of frequency 3.00 MHz. If the attenuation of this wave is 3.8 dB/cm as it propagates into muscle tissue, at what depth (measured in millimetres) is 50% of the energy delivered?

PROBLEM 5.2.06: An ultrasound device is set to produce waves of frequency 1.00 MHz. If the attenuation of this wave is 4.95 dB/cm as it propagates through tendon, at what depth (measured in millimetres) is 37% of the energy delivered?

PROBLEM 5.2.07: In the human body sound waves travel at about 1540 m/s. An ultrasound device is set to produce waves of frequency 4.00 MHz.
(a) What is the wavelength (in millimetres) of these waves, in the body?
(b) If the attenuation of the wave is 8.2 dB/cm, at what depth (measured in millimetres) is 90% the energy delivered?
(c) What is this depth, expressed as a multiple of the wavelength?

PROBLEM 5.2.08: The attenuation coefficient of skin at 2 MHz is approximately 5 dB/cm. If 2 MHz ultrasound of intensity 0.64 W/cm^2 is applied over an 12 cm-diameter circular area of skin that is 3 mm thick:
(a) What is the intensity of the wave just below the skin?
(b) What amount of energy is delivered into the portion below skin after seven minutes?

Chapter 6

Electricity

"Electricity" is broad term for the phenomena related to the *controlled* transfer of electric charge. In the context of *electric circuits* this allows for the controlled transfer and transformation of *electric energy*.

6.1 Electric Current, Voltage and Power

Electric charge changing location transfers electric energy. Interaction of these charges with their surrounding material can then transform their electric energy into other forms. With current measuring the rate of charge motion it becomes meaningful to speak of the power transferred by current. The following exercises explore these ideas.

6.1.1 Current, Charge and Time

Electric ***charge*** is an intrinsic physical property (like mass) of the fundamental particles of matter.

Electric ***current*** measures the rate at which electric charge moves from place to place.

$$I = \frac{|\Delta Q|}{\Delta t} \tag{6.1}$$

With charge measured in coulombs and time in seconds, current is measured in *amperes* (1 A=1 C/s). Since a coulomb is a large amount of charge we will quite frequently be measuring currents in smaller increments such as milliamperes mA and microamperes μA. Notice that this quantity depends upon the *magnitude* of the charge, and not its sign.

Because of the relation between charge, time and current, an alternate unit for charge is the millampere-hour ($1\,\text{mAh} = 0.001\,\text{A} \times 3600\,\text{s} = 3.6\,\text{C}$). This unit is often used for measuring the capacity of small batteries, like those found in portable electronics.

Exr 6.1.01 What is the electric current (measured in milliamperes) when 75.0 C of charge is transferred in 120 s?

Exr 6.1.02 What is the electric current (measured in microamperes) when 632 μC of charge is transferred in 7.02 s?

Exr 6.1.03 What is the electric current (measured in amperes) when $37.0 \times 10^{+12}$ electrons are transferred in 1.00 s?

Exr 6.1.04 What is the electric current (measured in amperes) when one-tenth of a mole of electrons are transferred in one hour?

Exr 6.1.05 What is the electric charge (measured in coulombs) transferred when a current of 0.500 A flows for 12.0 s?

Exr 6.1.06 What is the electric charge (measured in coulombs) transferred when a current of 128 mA flows for seven and a half minutes?

Exr 6.1.07 What is the electric charge (measured in milliamp-hours) transferred when a current of 88.8 mA flows for 20 minutes and 16 seconds?

Exr 6.1.08 If a current of 210 mA flows for 7 minutes how many moles of electrons are transferred?

Exr 6.1.09 What amount of time (measured in minutes and seconds) must elapse for a 1.30 A current to transfer 420 C?

Exr 6.1.10 What amount of time (measured in hours and minutes) must elapse for a 0.750 A current to transfer 1 200 mAh?

Exr 6.1.11 Car batteries have capacities measured in amp-hours (not *milli*amp-hours). What amount of time (measured in minutes and seconds) must elapse for a 833 A current to drain 70 Ah from a car battery?

6.1.2 Voltage, Charge and Energy

Similar to how there is gravitational energy between separated masses there is *electrical* energy between separated charges. The electrical potential difference, commonly referred to as the *voltage difference*, is defined by

$$\Delta V = \frac{\Delta E}{\Delta Q} \tag{6.2}$$

This is the ratio of the change in electrical energy to the amount of charge that experienced that change. (The analogous quantity in the case of gravity would be $(mg\,\Delta y)/m = g\,\Delta y$, which only depends upon the interaction and its geometry, but not the thing that is being interacted with.) The units of voltage difference are *volts*: $1\,\mathrm{V} = 1\,\mathrm{J/C}$.

As an example, if $-72\,\mathrm{mC}$ charge is transferred across a $+1.50\,\mathrm{V}$ difference, the change in electrical energy is $\Delta E = \Delta V \times \Delta Q = (+1.50\,\mathrm{V}) \times (-0.072\,\mathrm{C}) = -0.108\,\mathrm{J}$. It is ***critical*** to note that, unlike the definition of current, the *signs* of the quantities involved are important.

Exr 6.1.12 What voltage difference was crossed by $+0.700\,\mathrm{mC}$ if its electric energy changed by $+8.40\,\mathrm{mJ}$?

Exr 6.1.13 What voltage difference was crossed by $+0.700\,\mathrm{C}$ if its electric energy changed by $-77.7\,\mathrm{mJ}$?

Exr 6.1.14 What voltage difference was crossed by an electron if its electric energy changed by $-0.2755 \times 10^{-15}\,\mathrm{J}$?

Exr 6.1.15 What voltage difference was crossed by $+0.833\,\mathrm{mAh}$ if its electric energy changed by $-27.0\,\mathrm{J}$?

Exr 6.1.16 If $+2.11\,\mathrm{mC}$ crosses a voltage difference of $-0.570\,\mathrm{V}$ what is the change in the charge's electrical energy?

Exr 6.1.17 If $-73.5\,\mu\mathrm{C}$ crosses a voltage difference of $-4.44\,\mathrm{mV}$ what is the change in the charge's electrical energy?

Exr 6.1.18 If $-25.6 \times 10^{-12}\,\mathrm{C}$ crosses a voltage difference of $+5.08 \times 10^{+4}\,\mathrm{V}$ what is the change in the charge's electrical energy?

Exr 6.1.19 If $+295\,\mathrm{mAh}$ crosses a voltage difference of $+12.0\,\mathrm{V}$ what is the change in the charge's electrical energy?

Exr 6.1.20 What amount of charge needs to cross $640\,\mathrm{mV}$ to transfer $10.24\,\mu\mathrm{J}$?

Exr 6.1.21 What amount of charge (measured in mAh) needs to cross $1.500\,\mathrm{V}$ to transfer $15.12\,\mathrm{kJ}$?

6.1.3 Electrical Power

Charges moving across a voltage difference transfer energy. When current flows across a voltage difference, the rate of charge motion relates to the rate of energy transfer, which is *power*:

$$P = I \times \Delta V \tag{6.3}$$

Consider the units of current times voltage:

$$\mathrm{A} \times \mathrm{V} = \frac{\mathrm{C}}{\mathrm{s}} \times \frac{\mathrm{J}}{\mathrm{C}} = \frac{\mathrm{J}}{\mathrm{s}} = \mathrm{W} \tag{6.4}$$

Also, from the relation between power and energy, we have that

$$\Delta E = P \times \Delta t \tag{6.5}$$

$$= I \times \Delta V \times \Delta t \tag{6.6}$$

EXERCISE 6.1.22 What power is being delivered by 250 mA flowing through a 1.50 V AA-battery?

EXERCISE 6.1.23 What power is being delivered by 62 A flowing through a 12 V car battery?

EXERCISE 6.1.24 What power is being delivered by 2.15 A flowing through a resistor with a 5.52 V voltage difference across it?

EXERCISE 6.1.25 Charging a 9.00 V battery by forcing 0.987 A through it delivers what power?

EXERCISE 6.1.26 What energy is delivered by 325 mA flowing across 1.50 V after 30 s?

EXERCISE 6.1.27 What energy is delivered by 0.850 mA flowing across 33.3 mV after seventeen minutes?

EXERCISE 6.1.28 What current must flow across 120 V is required to deliver 1500 W?

EXERCISE 6.1.29 What current must flow across 9.00 V is required to deliver 7.20 W?

6.1.4 Problems

PROBLEM 6.1.01: A total of 720 mAh of electrons flows through a 1.500 V battery.
(a) What amount of charge in coulombs (including its sign) crossed the battery?
(b) If this charge *gained* electrical energy, what was the *sign* of the voltage difference?
(c) What was the change in the charge's electrical energy?
(d) If the battery was providing this energy at a constant rate of 300 mW, what was the current?

PROBLEM 6.1.02: Desk-top cup warmer:
(a) How many AA batteries would you need to warm a cup of tea by 3.12 C°? (Each 1.500 V battery can deliver 2 800 mAh. The cup contains 230 mL of tea, which has a heat capacity roughly that of water.)
(b) If we want to achieve this warming in five minutes, what must be the resistance of the heating element?

6.2 Electric Circuits

Electric charge changing location (a *current*) transfers electric energy. Interaction of these charges with their surrounding material can then transform their electric energy into other forms, and vice versa. An ***electric circuit*** is an arrangement of circuit elements (like batteries, resistors, LEDs, etc) connected to each other by wires.

6.2.1 Ohm's Law & the Simple Circuit

Ohm's Law relates the difference in electrical potential (the voltage difference ΔV) across a resistor (R measured in ohms) to the current (I measured in amperes) that flows through it:

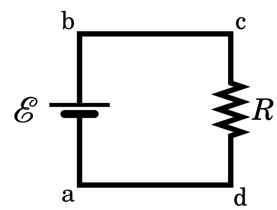
$$\Delta V = IR \tag{6.7}$$

The unit of resistance is the ohm: $1\,\text{ohm} = 1\,\Omega = 1\,\text{V/A}$. Since electrical power is given by $P = I\,\Delta V$, we also have

$$P = I^2 R \tag{6.8}$$

as the rate at which electric energy is dissipated as thermal energy by a resistor. This relation also shows that one ohm is equivalent to $1\,\Omega = 1\,\text{W/A}^2$. (Algebra can also show that $P = (\Delta V)^2/R$.)

The simplest circuit is a battery and a resistor connected in a single loop, as shown in the diagram below:



The voltage difference sustained by a battery (or generator, or other source of electrical energy) is referred to as the *electro-motive force*, or *emf* for short, denoted by the symbol $\mathcal{E}$. The value of the emf does not depend upon (or vary with) what it is connected to in the circuit. What does vary, and depends upon the other elements in the circuit, is the current that flows through the battery.

EXR 6.2.01 If the emf of the battery is $\mathcal{E} = 5.00\,\text{V}$ and the resistance in the circuit is $R = 220\,\Omega$, then find
(a) the current flowing around the circuit,
(b) the power dissipated by the resistor.

EXR 6.2.02 If the emf of the battery is $\mathcal{E} = 17.4\,\text{V}$ and the resistance in the circuit is $R = 421\,\Omega$, then find
(a) the current flowing around the circuit,
(b) the power dissipated by the resistor.

EXR 6.2.03 If the emf of the battery is $\mathcal{E} = 20.0\,\text{V}$ and the current flowing around the circuit is $I = 5.00\,\text{A}$, then find
(a) the value of the resistance,
(b) the power provided by the battery.

EXR 6.2.04 If the emf of the battery is $\mathcal{E} = 8.37\,\text{V}$ and the current flowing around the circuit is $I = 792\,\text{mA}$, then find
(a) the value of the resistance,
(b) the power provided by the battery.

EXR 6.2.05 If the emf of the battery is $\mathcal{E} = 8.00\,\text{V}$ and it provides $P = 16.0\,\text{W}$ of power, then find
(a) the current around the circuit,
(b) the value of the resistance.

EXR 6.2.06 If the emf of the power supply is $\mathcal{E} = 120\,\text{V}$ and it provides $P = 1.200\,\text{kW}$ of power, then find
(a) the current around the circuit,
(b) the value of the resistance.

EXR 6.2.07 If the resistance is $R = 220\,\Omega$ and a current $I = 33.2\,\text{mA}$ flows around the circuit, then find
(a) the emf of the battery,
(b) the power dissipated by the resistor.

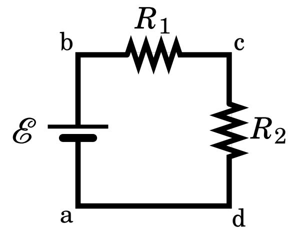
EXR 6.2.08 If the resistance is $R = 33.33\,\Omega$ and a current $I = 3.60\,\text{A}$ flows around the circuit, then find
(a) the emf of the battery,
(b) the power dissipated by the resistor.

6.2.2 Circuits with Resistors in Series

In *series*, the current through each element of the circuit (the battery, and each of the resistors) is the same. When resistors are connected in series the *equivalent resistance* of the resulting circuit is given by

$$R_{\text{eq}} = R_1 + R_2 \tag{6.9}$$

The current that flows through the battery, and each of the resistors, is given by $I = \mathscr{E}/R_{\text{eq}}$.



EXERCISE 6.2.09 If the emf of the battery in the series circuit is $\mathscr{E} = 5.00\,\text{V}$ and the resistances are $R_1 = 220\,\Omega$ and $R_2 = 150\,\Omega$, then find
(a) the current flowing around the circuit,
(b) the voltage difference across the small resistor,
(c) the power dissipated by the smaller resistor.

EXERCISE 6.2.10 If the emf of the battery in the series circuit is $\mathscr{E} = 16.3\,\text{V}$ and the resistances are $R_1 = 53.0\,\Omega$ and $R_2 = 17.0\,\Omega$, then find
(a) the current flowing around the circuit,
(b) the voltage difference across the small resistor,
(c) the power dissipated by the smaller resistor.

EXERCISE 6.2.11 If the emf of the battery in the series circuit is $\mathscr{E} = 70.0\,\text{V}$, the resistance $R_1 = 98.0\,\Omega$, and the current flowing around the circuit is $I = 345\,\text{mA}$, then find the value of the unknown resistor.

EXERCISE 6.2.12 If the emf of the battery in the series circuit is $\mathscr{E} = 120\,\text{V}$, the resistance $R_1 = 34.3\,\Omega$, and the current flowing around the circuit is $I = 1.75\,\text{A}$, then find the value of the unknown resistor.

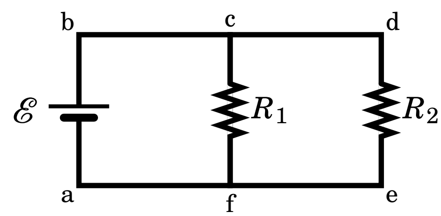
EXERCISE 6.2.13 If the emf of the battery in the series circuit is $\mathscr{E} = 1.50\,\text{V}$, the resistance $R_1 = 1.22\,\Omega$, and the current flowing around the circuit is $I = 551\,\text{mA}$, then find the value of the unknown resistor.

6.2.3 Circuits with Resistors in Parallel

In *parallel*, the voltage across each element in the circuit (the battery, and each of the resistors) is the same. When resistors are connected in parallel the equivalent resistance of the resulting circuit is given by

$$\frac{1}{R_{\mathrm{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{6.10}$$

Your must remember to take the reciprocal of this!: $R_{\mathrm{eq}} = (1/R_{\mathrm{eq}})^{-1}$. The current that flows through the battery is given by $I = \mathscr{E}/R_{\mathrm{eq}}$.



EXERCISE 6.2.14 If the emf of the battery in the parallel circuit is $\mathscr{E} = 25.0\,\mathrm{V}$ and the resistances are $R_1 = 11.0\,\Omega$ and $R_2 = 22.0\,\Omega$, then find
(a) the equivalent resistance of the circuit,
(b) the current out of the battery,
(c) the current through each resistor.

EXERCISE 6.2.15 If the emf of the battery in the parallel circuit is $\mathscr{E} = 75.0\,\mathrm{V}$ and the resistances are $R_1 = 150\,\Omega$ and $R_2 = 68.0\,\Omega$, then find
(a) the equivalent resistance of the circuit,
(b) the current out of the battery,
(c) the current through each resistor.

EXERCISE 6.2.16 If the emf of the battery in the parallel circuit is $\mathscr{E} = 1.50\,\mathrm{V}$ and the resistances are $R_1 = 0.375\,\Omega$ and $R_2 = 0.920\,\Omega$, then find
(a) the equivalent resistance of the circuit,
(b) the current out of the battery,
(c) the current through each resistor.

6.3 Time-Varying Currents

When electric current flows charges move through each element of the circuit. In the context of your *Electrotherapy* course a portion of the patient's body is an element of the circuit. In the human body it is a mixture of charged molecules, both positive and negative, that move when electric current flows in the body. With the boundaries of cells, organs, and ultimately the skin, electric current flowing in the human body has nowhere to go. Thus a current that flows at a constant rate through the human body will separate the charged molecules – with the positively charged molecules migrating to one side and the negatively charged molecules migrating to the other.

One method to avoid separating the different charges, while still permitting current, is alternate the direction of current flow. (Think of this like when you rub your hands together to produce heat by friction, and you alternate the direction of your hand's motion.)

Another method is to periodically decrease the applied voltage to zero. This gives the partially separated charges time to move back together and recombine into its original neutral mixture. That process is called *relaxation*.

Although the current may vary with time, all the relations between current, voltage and resistance still apply, since they apply *at each instant in time*. The same is true of the relation between current, voltage and power. So these relations are true at each instant in time:

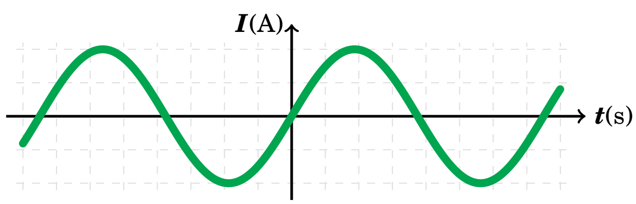
$$\Delta V = I \times R \tag{6.11}$$

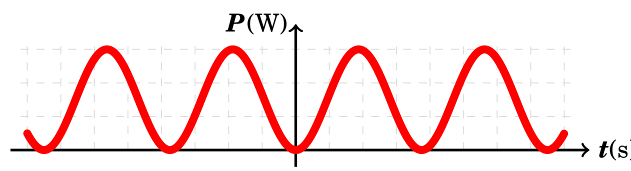
$$P = I \times \Delta V \tag{6.12}$$

What is complicated now are the relations between current and charge transported, and between power and energy transferred or transformed. These *totals* will now depend upon exactly how the current varies with time, and will often be expressed as *averages* over intervals of time that are large in comparison to the changes.

6.3.1 Alternating Current

One of the fundamental types of time-varying current is referred to as *Alternating Current*, which is abbreviated "AC". The current (and voltage) vary sinusoidally. (This is the type of current provided by electrical wall outlets.) If the current varies as the sine-function, then (as an example) the *instantaneous* power being dissipated by a resistor ($P = RI^2$) will vary as the square of the sine-function. This is graphed below:





The rate of energy dissipation does not depend upon the *direction* of charge motion, only its rate. So even as the current reverses direction – in the graph, where the current takes negative values over half of each cycle – the power is same as in the first half of the cycle. Calling the amplitude of the current oscillation I_{peak}, the maximum value of the power is $P_{\text{peak}} = R\,(I_{\text{peak}})^2$.

Averaged over many cycles the average power being dissipated is half of the peak value:

$$P_{\text{avg}} = \tfrac{1}{2} P_{\text{peak}} \tag{6.13}$$

This is defined by the fact that the total energy ΔE delivered by the alternating current will be $\Delta E = P_{\text{avg}}\,\Delta t$, where Δt is the total time that the current is oscillating. (This can be proved rigorously using calculus.)

This relation between the peak and average power is used to define a measure of the "average current". (Strictly speaking, since the current alternates direction, the current averages out to *zero*. The "average" being defined here is more like a measure of the *amplitude* of the oscillating current.) Defining I_{rms} the *root mean square* (RMS) average of the current through $P_{\text{avg}} = R\,(I_{\text{rms}})^2$, we obtain

$$P_{\text{avg}} = \tfrac{1}{2} P_{\text{peak}} \tag{6.14}$$

$$R\,(I_{\text{rms}})^2 = R\,(I_{\text{peak}})^2 \tag{6.15}$$

$$I_{\text{rms}} = \tfrac{1}{\sqrt{2}} I_{\text{peak}} \tag{6.16}$$

Since $\frac{1}{\sqrt{2}} \approx 0.7$ the RMS (average) value of the current is about 70% of the peak value. It is important to have these two measures of the alternating current because: (1) the average of the power relates to the target energy we seek to deliver; but (2) the peak current is important to control for safety reasons.

Through Ohm's Law the RMS (average) value of the oscillating voltage difference that drives the oscillating current is similarly defined in relation to its peak value:

$$\Delta V_{\text{rms}} = \frac{1}{\sqrt{2}} \Delta V_{\text{peak}} \tag{6.17}$$

(If you are aware of it, the "120 V" of the electrical supply from wall plugs is the RMS value of the oscillating voltage.) When oscillating voltages and currents are measured using "multimeters" it is usually the RMS values being reported. These relations hold:

$$\Delta V = I \times R \tag{6.18}$$

$$\Delta V_{\text{peak}} = I_{\text{peak}} \times R \tag{6.19}$$

$$\Delta V_{\text{rms}} = I_{\text{rms}} \times R \tag{6.20}$$

This first is true at each instant in time. The second relates the values at the instant the current is at its peak. The third relates the values when averaged over an interval of time that is long in comparison to the period of oscillation.

Frequency Dependence

The *expressions* for Ohm's Law ($\Delta V = I R$) and the instantaneous power ($P = I \, \Delta V$) do not depend upon the frequency of the current's alternation. What does vary with frequency, usually, is the resistance of the material, typically increasing with frequency – although the truth is quite complicated, and there are many exceptions. In practice, the frequency will usually be prescribed and the resistance under those conditions either known, or easily determined.

The resistance may vary with frequency, but the peak voltages and currents are independent of the frequency. Just remember that, as we saw for *waves*, amplitude and frequency are independent of each other.

Exr 6.3.01 If an alternating current $I_{rms} = 13.0\,\mathrm{mA}$ flows through a $220\,\Omega$ resistor, find
(a) the RMS voltage being applied,
(b) the peak current flowing through the resistor, and
(c) the average power being dissipated.

Exr 6.3.02 If an alternating current $I_{rms} = 15.0\,\mathrm{A}$ flows through a $8.00\,\Omega$ resistor, find
(a) the RMS voltage being applied,
(b) the peak current flowing through the resistor, and
(c) the average power being dissipated.

Exr 6.3.03 If an alternating voltage difference $\Delta V_{rms} = 120\,\mathrm{V}$ is applied across a $68.0\,\Omega$ resistor, find
(a) the RMS current flowing through the resistor,
(b) the peak current flowing through the resistor, and
(c) the average power being dissipated.

Exr 6.3.04 If an alternating voltage difference $\Delta V_{rms} = 9.00\,\mathrm{V}$ is applied across a $725\,\mathrm{m\Omega}$ resistor, find
(a) the RMS current flowing through the resistor,
(b) the peak current flowing through the resistor, and
(c) the average power being dissipated.

Exr 6.3.05 If an alternating voltage difference $\Delta V_{rms} = 75.0\,\mathrm{V}$ dissipates $210\,\mathrm{W}$ (average) in a resistor, find
(a) the RMS current flowing through the resistor, and
(b) the value of the resistance.

Exr 6.3.06 If an alternating voltage difference $\Delta V_{rms} = 120\,\mathrm{V}$ dissipates $15.3\,\mathrm{W}$ (average) in a resistor, find
(a) the RMS current flowing through the resistor, and
(b) the value of the resistance.

Exr 6.3.07 If an alternating voltage difference $\Delta V_{rms} = 120\,\mathrm{V}$ is applied across a $22.2\,\Omega$ resistor, find the time required to dissipate $7.31\,\mathrm{kJ}$.

Exr 6.3.08 If an alternating voltage difference $\Delta V_{rms} = 5.00\,\mathrm{V}$ is applied across a $417\,\mathrm{m\Omega}$ resistor, find the time required to dissipate $29.2\,\mathrm{kJ}$.

